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Numerical solution of the gentle slope equation and its application in the design of the protection works of a marina in Nayarit Mexico

Solución numérica de la ecuación de la pendiente suave y su aplicación en el diseño de las obras de protección de una marina náutica en Nayarit México

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Abstract

Numerical solution of the mild-slope equation and its application in the design of the breakwater structure of yachting harbor in Nayarit Mexico. We present the numerical solution of the Berkhoff method (1971) known as the mild slope equation in elliptical shape for deep water and similarly for shallow areas considering the speed of wave group depending on the depth and the acceleration of gravity occurs. The equation has been subdivided into three equations simultaneously to calculate the direction, wave height and phase of the wave; solving the system of equations is determined by a numerical method a relaxation algorithm for calculating the phase of the wave. Finite difference scheme with elliptical approach and staggered- celd with the option of a fine mesh in areas where protection works or structures that modify the surf for study are used. The numerical calibration was performed with simulation examples of literature in its analytically presented examples of application with constant conditions obtaining acceptable values (Herrera, 2009). The application of the model was carried out in protection structures a yachting harbor located in Guayabitos Nayarit where we need to know the magnitude of the agitation of the free surface generated by wave transmission considering the projected yachting harbor geometry passing through the mouth. The results of modeling propose the type and dimensions of protection structures required to minimize the agitation conditions within the yachting harbor.

Numerical modeling, Ocean wave, Protection structures

Resumen

Se presenta la solución numérica de la ecuación de Berhkoff (1971) conocida como la pendiente suave en su forma elíptica para profundidades indefinidas y análogamente para zonas someras considerando la celeridad de grupo de ola en función de la profundidad y de la aceleración de la gravedad. La ecuación se ha subdividido en tres ecuaciones para calcular de forma simultánea la dirección, altura de ola y fase de la ola; la solución del sistema de ecuaciones es por un método numérico determinado con un algoritmo de relajación para el cálculo de la fase de la ola. Se emplea un esquema de diferencias finitas con aproximación elíptica y un mallado tipo staggered-celd con la opción de tener un refinado de malla en zonas donde se encuentren obras de protección o estructuras que modifique el oleaje para su estudio. La calibración se realizó con la simulación de ejemplos de literatura en su forma analítica que presentan ejemplos de aplicación con condiciones constantes obteniendo valores aceptables (Herrera, 2009). La aplicación del modelo se llevó a cabo en las obras de protección de una marina náutica ubicada en el Guayabitos Nayarit donde se necesita conocer la magnitud de la agitación de la superficie libre generada por la transmisión del oleaje considerando la geometría proyectada para la marina náutica a su paso por la bocana. Los resultados obtenidos de la modelación proponen el tipo y dimensiones de las obras de protección requeridas para minimizar las condiciones de agitación dentro de la marina.

Modelación numérica, Oleaje, Obras de protección

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Introduction

The interaction of waves with the seabed is one of the reasons why waves change. As a consequence of this interaction of waves with the seabed, the wave train or swell is transformed giving rise to several coastal processes visible to an observer from the shore. These transformation processes are mainly translated into: variation in wave height and in the direction of propagation. These are called refraction, diffraction, reflection and breaking.

Each of these phenomena has been analysed separately using graphical and numerical methods. It should be noted that in nature all the phenomena are related to each other, therefore, in the present work we will describe the development of a numerical model that takes into account several phenomena and its application to a study area whose main problem is to know the agitation or oscillation of the free surface caused by the incident waves.

The numerical model developed on the solution of the Berkhoff equation (1972) which, by means of a finite difference scheme with Matlab programming, can model regular waves in regions with irregular bathymetry. The discretisation of the equations, numerical solution and examples of validation of the model can be consulted in the work of Herrera (2009).

Materials and methodology

The equation developed by Berkhoff (1972), also known as the gentle slope equation, is one of the equations that works very well to simulate the refraction-diffraction-reflection phenomena, in places where the irregularity of the bottom would cause energy concentration during the advance of the wave front. To understand this equation, it is presented in its general form by defining each of its parts as follows:

$$\underbrace{\frac{\partial E(x,y,t,f,\theta)}{\partial t}}_{1} + \underbrace{\nabla \cdot \left[C_y(x,y,f) * E(x,y,t,f,\theta)\right]}_{2} = \underbrace{S_w}_{3} + \underbrace{S_m}_{4} + \underbrace{S_d}_{5} + \underbrace{S_f}_{6} + \underbrace{S_p}_{7} \qquad (1)$$

Where the first term represents the rate of temporal change of the spectrum, the second term represents the propagation of wave energy, the third term represents wind inputs, the fourth term represents the redistribution of wave energy among various non-linear components that occur, the fifth term represents dissipation due to the breaker, the sixth term represents losses due to friction, and the seventh term represents losses due to seepage.

Numerical modelling is used to solve eq. (1) in an elliptic approximation (eq. 2) that describes the propagation of a periodic, finite-amplitude free surface wave over complex bathymetries, where its deformation on approaching shallow depth and obstacle zones exhibits the phenomena of refraction, diffraction and reflection (Panchang *et al.*, 1991).

$$\nabla\left(C_{Cg}\nabla\eta\right) - \frac{C_g}{C}\frac{\partial^2\eta}{\partial t^2} = 0$$

Where:

η(x,y) is the level or elevation of the free surface (m).

C(x,y) phase velocity or phase velocity (m/s)

Cg(x,y) Group velocity (m/s)

The solution of the Berkhoff equation is worked out and expressed as a function of the wave flow ratio in its x and y components (Fuentes, 1996), this by means of a scheme implicit in time by means of the following finite difference equations.

$$c^2\frac{\partial\eta}{\partial x} = -\frac{\partial Q_x}{\partial t}$$

$$c^2\frac{\partial\eta}{\partial y} = -\frac{\partial Q_y}{\partial t}$$

Where Q (x, y) is the flow ratio in the horizontal plane (m/s).

For the solution of the equation for the variation of the free surface area due to waves as a function of the above-mentioned flow ratios we have:

$$\frac{\partial\eta}{\partial t} = -\frac{1}{n}\left[\frac{\partial}{\partial x}(nQ_x) + \frac{\partial}{\partial y}(nQ_y)\right]$$

$$n = \frac{1}{2}\left(1 + \frac{2kh}{senh(2kh)}\right)$$

Where:

n (x, y) Group factor

k (x, y) Wave number (2π/L)

h (x, y) Depth (m)

This type of equations has the advantage of having the values of the flux ratios and flow and that they allow us to find the direction of the and to consider the presence of an obstacle in a simple way.

Boundary conditions

To obtain the wave propagation in the free boundaries, Snell's law is applied, which allows the wave to leave the domain of study without modifying its direction or magnitude; the obstacles or structures within the domain are considered solid boundaries, so these are proposed as a totally vertical wall, this wall presents 3 reflection conditions which are manifested with a coefficient of (KL), whose value is limited between 0 and 1 (0 for a structure that does not present reflection, 0.5 for a partially reflecting structure and 1 for a totally reflecting structure, 0.5 for a partially reflecting structure and 1 for a totally reflecting structure). fully reflective condition). This applies around the perimeter of the structures.

In order to obtain the elevation of the following equations are used to obtain the elevation of the free surface at the ends of any obstacle:

$$\eta_0^{n+1} = (1 + K_L) * \eta^n_{\left(\frac{c\Delta t}{\Delta x}\right)} - K_L \eta_0^{n-1}$$

$$\eta_L^{n+1} = (1 + K_L) * \eta^n_{\left(L-\frac{c\Delta t}{\Delta x}\right)} - K_L \eta_0^{n-1}$$

Eq. (7) is used for the start of obstacle with infinite length, and Eq. (8) is used for the end of the obstacle is used for the end of the obstacle. For the calculation of a quasi-oscillatory wave, it results from the superposition of an incident wave with height (Hi) and a wave travelling in the opposite direction with a lower height (Hr). This reduction in wave height and the phase lag between incident and reflected are associated and the resulting surface can be expressed:

$$\eta_{i,j}^{n+1} = \frac{H_{i,j}}{2} cos\left[\left(Kx_{i,j}X_{i,j}cos\left(\theta_{i,j}\right)\right) + \left(Ky_{i,j}Y_{i,j}sen\left(\theta_{i,j}\right)\right) - \frac{2\pi}{T} * t\right]$$

Where the wave reflected by any obstacle is considered to leave the study region.

Finally, the wave angle or direction can be estimated from the relationship between the phases (Qx^{n+1}) and (Qy^{n+1}):

Model validation

To perform the validation of the wave modulus, domains similar to the theoretical examples in the literature were designed to reproduce the phenomena of refraction, diffraction and reflection, determining wave heights, angles of incidence and comparisons between the analytical solutions and the modelling results.

Resonance in a rectangular harbour

As a first validation case, the problem of resonance in a rectangular harbour (fig. 1), whose analytical solution was presented by Unluate et al. (1973) and the numerical solutions presented by Maa et al. (1997) and Lee (1971), was taken as a first validation case. The geometry of the harbour is 0.3212 m long by 0.0605 m wide and a depth of 0.2576 m; the wave incidence angle is zero degrees and the wave height is 0.01m; the modelling parameters can be seen in table (1).

Parámetros	
H(m)	0.01
T(s)	Varía
θ (grados)	0.00
h(m)	0.2576
Δx (m)	0.10
Δy (m)	0.10
W x L (m)	4.497 x 1.845
MP x NP	45 x 20
Tiempo de cómputo (s)	41
W, L son el ancho y largo del dominio de estudio	
MP, NP, son los numéros de celdas en la dirección "x" y "y" respectivamente	

Table 1 Parameters used for wave model validation

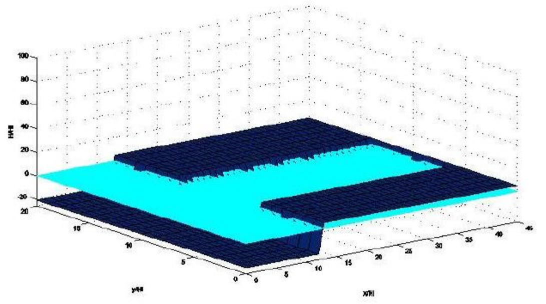

Figure 1 Geometry of the harbour

The domain was designed with 45 x 20 cells using Δx = Δy = 0.1 was compared with the solution obtained by Maa et al. (1997), which is presented in figure (2) the normalised wave heights.

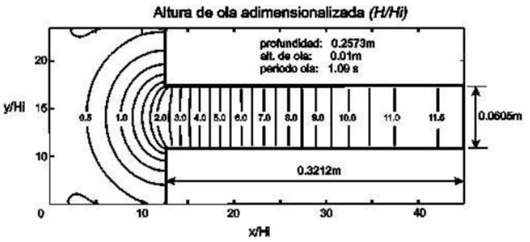

Figure 2 Solution presented by Maa *et al*., (1997)

Figure (3) shows the result obtained by the numerical model developed, where it was considered that all the walls have a reflection coefficient equal to 1.0, which is a totally reflective condition.

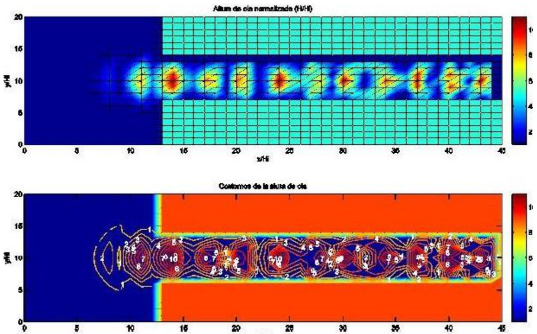

Figure 3 Results of the numerical simulation

Subsequently, a quantitative comparison was made between the solutions obtained by Lee (1971) and Maa et al. (1997), with reflection coefficients from 0 to 4 of the resonance presented by the harbour with different periods of incident waves. Figure (4) shows the results of the model with a relative percentage error between the values of the analytical solution and the calculated average values of 3.75%, due to the fact that the reflection on the faces perpendicular to the direction of the waves is considered, which presents a small agitation and is closer to reality.

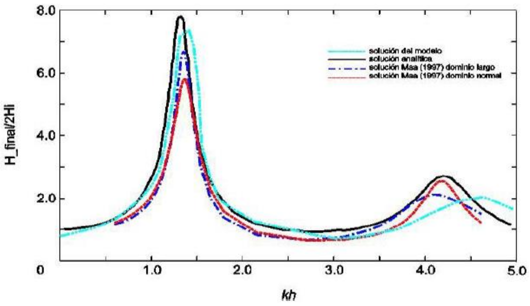

Figure 4 Comparison between different solutions of the harbour resonance analysis

Results and Application

The marina under study is located in the south of the state of Nayarit in the municipality of Compostela in the town of Los Ayala (fig. 5).

Figure 5 Location of the marina

The purpose of the study is to model the agitation that originates inside the marina due to the effects of the incident waves with NW direction, which is the predominant one in the region. In order to monitor the different levels of agitation that occur inside the marina and in some external points of interest, 10 control points or viewfinders were located, which allow a more specific diagnosis. Figure (6) shows a diagram of the marina with the location of the viewfinders.

Figure 6 Location of the control points or viewfinders during the simulations

The wave propagation data such as height, period and angle of incidence in the study domain are known and are supplied to the numerical model; so that the study grid (fig. 7) presents 145 x 148 cells with constant spacing for both directions of 5 mts.

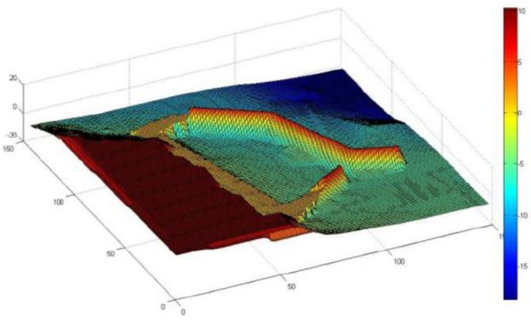

Figure 7 Numerical mesh for the simulation of the nautical marina

For the simulations of the incident waves on the protection works of the marina and their propagation within the marina, 4 wave scenarios are proposed, one under normal conditions and the others under extreme conditions.

1. Normal swell in a NW direction.
2. Hurricane waves in a NW direction with a return period of 10 years.
3. Hurricane storm surge in NW direction with a return period of 15 years.
4. Hurricane storm surge in NW direction with 20-year return period.

Simulation of agitation in the interior of the Marina with normal waves

According to the refraction-diffraction and wave height analysis, the waves to which the marina is most vulnerable under normal conditions are those coming from the north-west (NW) direction due to three main factors: the frequency of their presence, the way they arrive in the study area and the wave height, the marina is morphologically protected. Figure 8 shows the agitation inside the marina in the scenario described (NW direction, H = 3.21 m and T = 8.5 s).

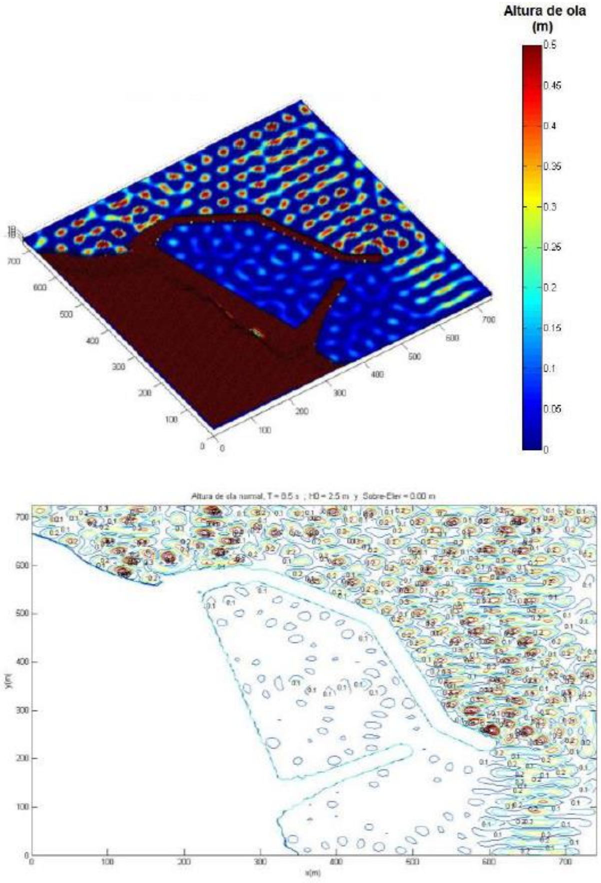

Figure 8 Disturbance inside the marina with a NW swell

Figure (8) shows the different wave heights inside the marina, using a colour scale ranging from zero in blue to 0.5 m in brown. In the interior of the marina we identify dark blue as the predominant colour with some lighter touches, which expresses a height of less than 0.10 m, and only in some areas of the marina is the colour of the waves. 0.10 m and only at some points is this height exceeded but without exceeding 0.20 m wave height, generating an area of great calm.

Figure (9) shows the wave heights of the control points where the wave height variations that exist inside the marina can be clearly observed. At points 1 and 2 we can observe relatively higher heights in relation to the rest of the points, produced by the reflection effects; however, their heights are of the order of 0.14 and 0.16 m respectively. 0.16 m respectively, wave heights that do not represent any risk to navigation or the mooring of vessels. Between points 3 to 8, none of the wave heights exceed 6 cm, which means that the morphological protection of the study area, as well as the arrangement of the proposed breakwaters, satisfies navigability conditions under normal climatic conditions.

Points 9 and 10 present an increase in wave height, this effect is defined because the location of these points is outside the protection works, however, their values do not exceed 15 cm in height, it is important to identify these values, as they correspond to the access mouth of the marina.

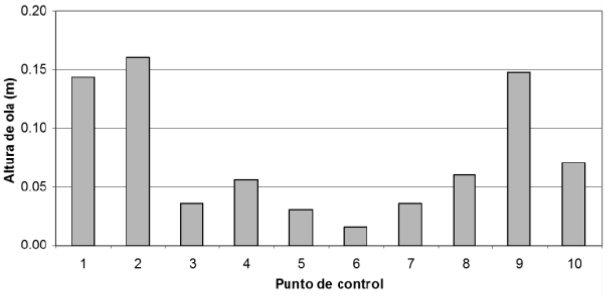

Figure 9 Wave heights determined at the control points for normal waves

Simulation of agitation inside the marina with hurricane waves

For the simulation of hurricane waves, different return periods (10, 15 and 20 years) were considered, for which the over-elevation generated in the study area was also estimated; the following figures show the results obtained.

10-year return period

For hurricane wave conditions, the heights generated in the deep-water region generally exceed the navigability conditions, however the proposed shelter to generate the calm zone inside the marina, significantly mitigates the agitation; in figure (10), the results obtained for a return period of 10 years, wave height H = 6.86 m, period T = 12 s and overelevation S = 1.42 m are shown. 1.42 m.

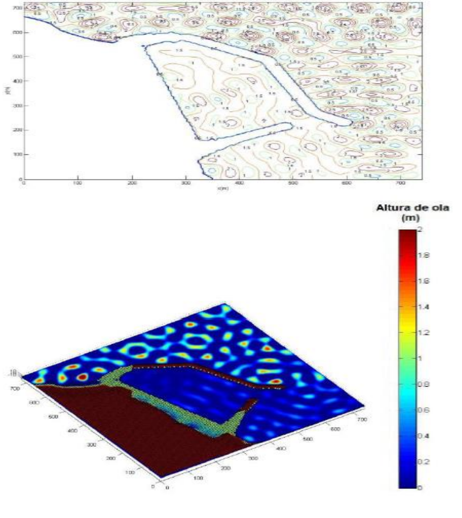

Figure 10 Perspective of the agitation inside the marina with hurricane waves, 10-year return period, NW wave direction, graphical scale from 0 to 2.0m.

Figure (10) shows the results obtained from the simulation carried out for a 10-year return period, where it can be seen that inside the marina, levels higher than 0.5m are not reached.

15-year return period

The results of the simulations for a return period of 15 years (wave height H=7.28 m, period T=12 s and over-elevation S=1.64 m), the agitation is presented in figure (11) using a colour scale that represents the wave heights on the same scale (0 - 2m) as in the simulation described above.

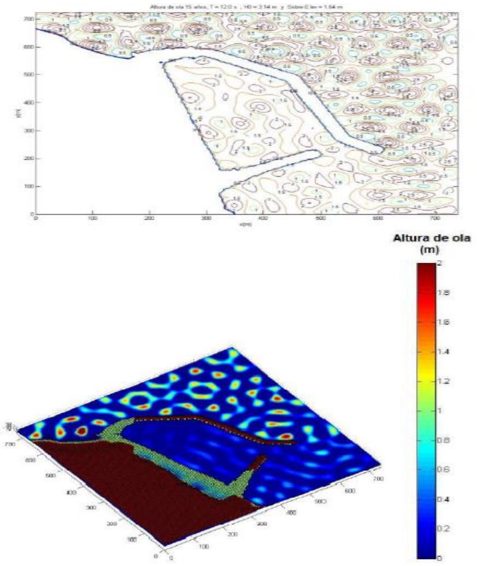

Figure 11 Perspective of the agitation inside the marina with hurricane waves, 15-year return period, NW wave direction, graphic scale from 0 to 2.0 m

Figure (11), shows a three-dimensional perspective view, showing that the protection provided by the proposed breakwaters, allows an agitation inside the marina that does not exceed 0.50 m, which reflects a favourable condition for the berthing of vessels, without endangering the integrity of the docks, nor that of the vessels. the integrity of the docks, nor that of the vessels.

20-year return period

This simulation represents the most unfavourable case, with a wave height of 7.57 m, a period of 12 s and an over-elevation of 1.79 m. The results are presented in figure (12), where it is observed that the wave agitation heights inside the marina are notably higher than the higher cases, however, they do not present a risky condition, the heights exceed 0.50 m (which in none of the cases is higher than 0.50 m). 50 m (which in none of the previous cases was exceeded), at some points, however, none exceed 0.70 m.

The area with the greatest agitation is the centre, as it is in this region where the waves and reflections from the different borders of the marina are concentrated. the marina.

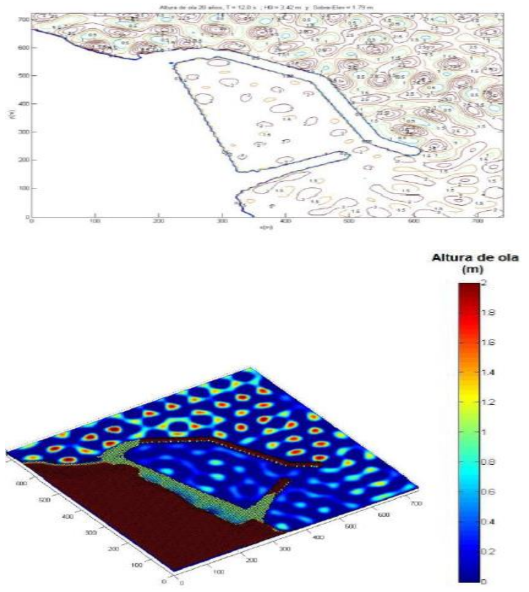

Figure 12 Perspective of the agitation in the interior of the marina with hurricane waves, return period of 20 years, NW wave direction, graphic scale from 0 to 2.0 m.

Figure (12), shows a three-dimensional perspective view, showing that the protection provided by the proposed breakwaters, allows for a turbulence inside the marina that does not exceed 0.70 m, even though this condition may present a moderate risk to the berthing of vessels, the frequency of presence of this turbulence is 20 years.

Conclusions

From the results described above, the following conclusions are drawn.

The arrangement of the protection works satisfactorily fulfils its function of providing minimum agitation inside the Marina basin for normal conditions.

For the extraordinary wave conditions, the agitation inside the marina does not present any risk for the 10- and 15-year return periods, however, for the 20 year return period the conditions can be dangerous for some vessels or for the marina facilities, so it is suggested to identify the areas of less agitation and direct the vessels present towards these areas, and also to prevent the mooring or navigation of vessels in the areas of greater agitation.

In terms of the operational limits within the marina, it is indicated that the agitation of the free surface may not exceed a wave height of 0.60 m in a period greater than 1.0 % per year.

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