

Empirical estimation of the volatility of the exchange rate in Mexico. A model of time series. 1995-2017

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Abstract

The present research aims to provide a relevant algorithm for the proper identification of a data generating process (economic and financial variables) by using the "auto.arima" command, belonging to the forecast library (R statistical software), to identify the optimal parameters of the ARIMA model. Considering the methodological framework of Box and Jenkins process (1970), it seeks to achieve, by solving a stochastic difference equation, the correct calibration and obtaining an optimal forecast for the Mexican Monetary Base, one of the main economic variables.

Forecast, Monetary Base, ARIMA Models

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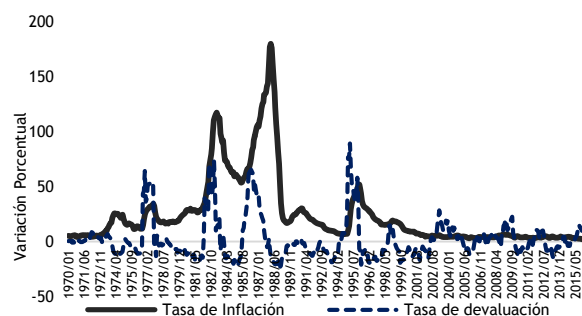
I. Introduction

In Mexico, reforms have been undertaken in favor of economic and financial liberalization with the objective of taking advantage of globalization. These reforms began in the 1980s with the adoption of the neoliberal model that included the deregulation and opening of financial markets and less State intervention. Throughout the 1990s, these reforms accelerated and deepened the process, which led the country to greater integration into the world economy.

In an increasingly international environment, advantages have been generated for countries, fostering an increase in foreign investment flows to developing countries, the reduction of tariffs on trade, the creation of large financial innovations (derivatives and securitization), others. But deep economic instability has also been fostered by being more exposed to external changes, the opening of markets has produced a sustained increase in both trade relations and in the circulation of capital, the problems of financial supervision have become much more complex and the handling of the exchange rate has become increasingly relevant because of its effects not only in the domestic economy but in the international financial system.

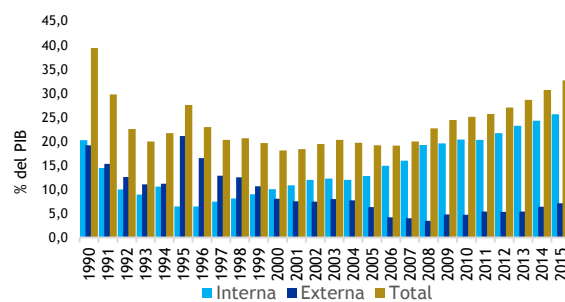
According to Caballero R. and Caballero (2016), fluctuations in the exchange rate have acted as a channel for contagion of international crises to countries, as the effects of these crises deepen when countries have a strong trade opening high pass-through of the exchange rate towards domestic prices (graph 1), and dollarization of its liabilities (graph 2). In this regard, Mexico exhibits all three characteristics, which indicates that exchange rate instability may continue to be a transmission mechanism, since exchange volatility continues to be an important element for any economic activity that involves international exchanges.

Whether physical assets and / or financial, in this sense remember the "Tequila Effect".



Graph 1 Magnified transfer of the exchange rate to inflation in Mexico. 1970-2015

Source: Own elaboration with data obtained from INEGI



Graph 2 Composition of public debt in Mexico (1990-2015)

Source: Own elaboration with data obtained from INEGI

There are several exchange rate regimes that the Mexican economy has experienced, however, it has been argued that exchange rate flotation systems can be more risky than fixed systems, since they imply greater volatility since they change continuously; This is because they are determined by their supply and demand, and are more exposed to structural changes and / or external shocks that cause uncertainty and which in turn affect investment decisions because they generate risks for companies and individuals (Ortiz et al, 2011). Some of the events that have impacted the volatility of the exchange rate are mentioned below:

- a. Devaluation in 1994: Mántey (2009) argues that a common denominator that gave rise to this crisis was mainly the maintenance of an exchange anchor that overvalued the Mexican peso excessively.
- b. Crisis of the Asian tigers 1997: Due to the strong speculative attacks, strong devaluations of the currencies were provoked, as a result the Asian countries set aside the fixed exchange rate regime to implement a free float exchange rate.
- c. Russian debt 1998: "The government and the central bank of Russia repudiated the payments on the internal debt, announced a moratorium on private external debits and abandoned the ruble to their fate" (Bracho, 2000). This caused the breakdown of most major banks, a sharp devaluation, the fall of the Stock Exchange, the interruption of all external capital, among others.
- d. Crisis of the Brazilian Real (1999): The fragility of Brazilian public finances and the moratorium declaration of the Gerais mines on January 6, 1999 caused the depletion of Brazilian reserves and the federal government had no alternative but to devalue.
- e. Crisis 2001: It was due to the deceleration of the North American economy through the so-called "dot com" crisis.
- f. Crisis of 2008: Low interest rates in the 2001-2006 period led to easy and cheap loans. The banks found new businesses in the conversion of consumer debt into marketable securities.
- g. The electoral triumph of Donald Trump (2016): After Trump's victory, the peso fell by more than 13%. It went from 18.5 per dollar to 20.74 per dollar. This is explained by the economic dependence so marked with the United States. With the entry into force of NAFTA, Mexico boosted the level of exports to its main trading partner (United States), but in turn increased its dependence on this country.

Thus, in an international context marked by the consequences of the financial and exchange crises and the fact that the exchange rate is one of the main channels for the transmission of inflation in developing countries, it has led to several central banks, mainly emerging economies to adopt inflation targeting (MI) schemes for the conduct of their monetary policy. These propose, among other aspects, that free market forces determine the exchange rate, where monetary policy is responsible for ensuring price stability by regulating aggregate demand through the interest rate and leaving aside the rate of interest change as nominal anchor.

However, although it has been argued that the exchange rate as a nominal anchor was left behind, this argument continues to be questioned, because although the formal instrument to impact on price stability is the interest rate, the The change in the Mexican peso has been key in the process of convergence of inflation (Mántey, 2009). Generally speaking, flexible exchange rates have led to an accelerated increase in real exchange rates and, as a result, have led to a consequent transfer of inflation to domestic prices, which has weakened investment and sustained economic growth.

This is the reason why the main Latin American economies seek at all costs to meet the objectives of inflation targets, and for that purpose it uses the exchange rate as a nominal anchor "(Rosas, 2016). This means an implicit recognition by the Bank of Mexico that the exchange rate is the main transmission channel of inflation; hence the "fear of floating" (Calvo and Reinhart, 2002).

The inflation targeting monetary policy scheme was used for the first time in 1990 by New Zealand and from then on, several countries, both developed and emerging, including Mexico in 2001, chose to adopt this scheme. Inflation goals "are a framework of monetary policy characterized by the public announcement of official quantitative targets for the rate of inflation in one or more time horizons, and by the explicit recognition that low and stable inflation is the main objective of the long-term monetary policy "(Hüfner, 2004). With the implementation of this model, the inflation rate in Mexico has effectively decreased (Figure 1), but it has left aside important aspects in the economy such as growth and unemployment.

Thus, the objective of this research paper is to analyze and estimate a measure of exchange rate volatility for Mexico in the period 1995-2017 with daily data using the family of GARCH models. This is due to the fact that exchange volatility continues to be a transcendental element for any economic activity that involves international exchanges. Also, the study period is interesting, since it involves periods of high exchange volatility accompanied by global economic and financial crises. The document is organized in 4 sections in addition to this introduction. The first refers to the review of the literature; the second shows the theoretical framework on the main stochastic models of volatility; in the third the econometric methodology is shown and in the last section the results are explained and the conclusions are presented.

2. Review of Literature

Since Engle in 1982 proposed to model the heteroscedastic nature of the time series using the ARCH models, the door was opened for the modeling of the exchange rate. Subsequently Bollerslev in 1986, due to the large number of parameters required by the ARCH models, raises the models known as GARCH.

The studies that analyze the transmission of volatility were initiated by Engle in (1990) and continued with Bollerslev (1990), Kearney and Patton (2000), Speight and McMillan (2001), among others. In all these studies, GARCH models were used to study the relationship between volatilities.

There are numerous studies that analyze the volatility of the exchange rate using GARCH models. In the case of Latin America, some studies focus on univariate models, namely, Grajales (2006), Carranza and González (2009) and Aranda, et. al in 2012. In 1998, Aragón and Atilano analyzed the effect of the type of regime in Mexico for the 1986-1998 period and concluded that in a flexible exchange regime, the estimation of a GARCH (1, 1) allows correctly model the volatility of the exchange rate, however its determinants must be analyzed to explain the violent changes and currency revaluations.

Viales (2010) uses historical GARCH series and concludes that the volatilities of Latin American countries are similar. On the other hand, Castaño, et al. (2006) present a work in which they calculate the term structure and the multi-period volatility of the Colombian peso, with the objective of knowing market expectations about the volatility of returns, in different time horizons. Aranda et. al (2012), compares various modeling alternatives, concluding that the GJR-GARCH model is the most adequate to forecast volatility at different horizons in Mexico.

Finally, although there are more studies, Bello in 2013 estimated the "value at risk" (value At Risk) from models GARCH, Zumaquero and Rivero (2010), analyze the structural changes and the behavior in the real exchange rate for a set of OECD countries during the period 1960-2006 in which they suggest that there is clear evidence in favor of the non-neutrality of the nominal exchange regime on the volatility of the real exchange rate for developed countries, but not in the case from developing or emerging countries.

In the case of Mexico, in 2009 Banco de México carried out a forecast of the volatility of the exchange rate and concluded that the best forecast is obtained when combining an ARCH forecast and an implied volatility forecast in options using weights that change over time. In 2012, Ortíz, et. they developed a forecast of exchange volatility using GARCH models, in which they evaluated the ability of two model specifications to forecast the volatility of the exchange rate between the Mexican peso and the United States dollar; with this they came to the conclusion that it is not possible to affirm that the assumption of normal errors is the best specification to efficiently forecast the exchange volatility. What can be said is that this specification could be adequate enough to be used for the purposes of managing foreign exchange risk.

In 2016, R. Caballero and Caballero analyzed the evolution of exchange rate volatility in Mexico and Brazil and presented evidence that it tends to decrease over time. In addition, they arrive at the following results: a greater exchange depreciation temporarily precedes a higher exchange rate volatility; once the volatility is in a certain regime (high or low), the possibility of passing to another regime immediately is very low, and finally, there is a high persistence of volatility in both economies (p.1).

3. Theoretical Framework

In 1970, Box and Jenkins amalgamated the ideas about the deterministic time series of Yule (1927) and introduced the stochastic component; these ideas formed the basis of a coherent and versatile approach that identified the iterative cycle of the series in three stages: identification, estimation and verification of the diagnosis. On the other hand, the volatility of the time series is an issue that for several decades has attracted the interest of researchers in the financial area. The seminal works of Engle (1982), Bollerslev (1986) and Taylor (1986) were the basis for the construction of a growing theoretical frame of reference for modeling conditional autoregressive heteroskedasticity.

In conventional econometric models, it is assumed that the variance of the error term is constant. However, many of the time series in the economy show periods of high volatility followed by relative tranquility. Under these circumstances, the assumption that there is a constant variance, that is to say that it is a homoscedastic series, is wrong. Given that the mean and the conditional variance have a lower bias than their non-conditional counterparts, they are preferable for the development of forecasts. The residuals of a GARCH model can come from an ARIMA model or a multiple linear regression model, so there are multiple possibilities for the construction of a hybrid model. Thus, the ARIMA and GARCH models are an essential tool for modeling the volatility of a very important economic variable, such as the exchange rate.

ARIMA Model

It is possible to combine a process of moving averages with a linear difference equation to obtain an autoregressive model and moving averages (ARMA). In these models it is perfectly permissible that the length of both processes be infinite. Consider the following representation:

$$Y_t = a_0 + \sum_{i=1}^p a_i Y_{t-i} + \sum_{i=0}^q B_i E_{t-i} \quad (1)$$

To ensure that an autoregressive econometric model is stable, that is, converges, the following rules must be met (Enders, 2015: 31).

1. In a n-th order equation, a necessary condition for all characteristic roots to fall within the unit circle is:

$$\sum_{i=1}^n a_i < 1$$

2. Since the value of a_i can be positive or negative, a sufficient condition for all characteristic roots to fall within the unit circle is

$$\sum_{i=1}^n |a_i| < 1$$

If one or more of the characteristic roots of (1) is greater than or equal to the unit, the sequence $\{Y_t\}$ is known as an integrated process and in this case the model is called Integrated Self-Regressive Moving Averages (ARIMA) (Enders, 2015:50). In this situation the time series becomes unstable, and its modeling can not be performed, because the function tends to infinity. This type of process is also known as "non-stationary". And some algebraic transformation must be applied that makes them stationary.

It is said that a series of time is stationary in a broad or weak sense, if and only if it satisfies three conditions:

- The average is constant over time.
 $E(Y_t) = E(Y_{t+T}) = \mu \quad \forall T \in \mathbb{R}.$
- The variance is constant over time.
 $Var(Y_t) = Var(Y_{t+T}) = \gamma \quad \forall T \in \mathbb{R}.$

- The autocorrelation function is independent of time. This measures the possible dependence between an observed value (Y_t) and other (Y_{t-k}) which is separated by a length interval k .

When the appropriate transformations have been made so that the time series is stationary, the development of the Box-Jenkins methodology can proceed. The ARIMA methodology, as it is also known, consists of the following stages:

- a. Identification of the Tentative Model. The simple and partial correlograms are used, with the purpose of determining a pattern of the different autocorrelations. In the same way, the correlogram is used to evaluate if there is some kind of seasonal variation, not eliminated by seasonal differences.
- b. Estimation of Parameters of the Model. For each tentative model the parameters must be estimated. This can be done using the ordinary least squares method, the Yule-Walker equation or the maximum likelihood method.
- c. Checking the diagnosis. Classic statistics should be calculated to measure the fit of the data using error measures and / or information criteria (AIC or BIC), the statistical significance of the estimated coefficients, unit root tests on the residuals of the model, and also must verify that there is no autocorrelation.

ARCH Model

Starting from a first order autoregressive stationary model:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t \quad (2)$$

When making a forecast, the conditional mean for the period y_{t+1} is:

$$E_t y_{t+1} = a_0 + a_1 y_t \quad (3)$$

While the conditional variance:

$$\text{var}(y_{t+1}|y_t) = E_t[(y_{t+1} - a_0 - a_1 y_t)^2] = E_t(\varepsilon_{t+1})^2 \quad (4)$$

Up to this point, it is established that $E_t(\varepsilon_{t+1})^2$ equals σ^2 , but by assuming that the conditional variance is not constant, it must be modeled as an AR (q) process using the squares of the estimated errors, so that:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + \dots + \alpha_q \hat{\varepsilon}_{t-q}^2 + v_t \quad (5)$$

Where v_t is a white noise process. If the values of $\alpha_1, \alpha_2, \dots, \alpha_n$ are equal to zero, the estimated variance is only a constant α_0 . Otherwise, the conditional variance of y_t evolves according to an autoregressive model. Thus, the conditional variance in the period $t+1$ is denoted as:

$$E_t \hat{\varepsilon}_{t+1}^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_t^2 + \dots + \alpha_q \hat{\varepsilon}_{t+1-q}^2 \quad (6)$$

For this reason, an equation like this one is called a model with conditional autoregressive heteroskedasticity (ARCH for its acronym in English). The residuals of said equation can come either from an autoregressive, an ARMA model or a standard regression model.

However, the specification of (5) is not the most convenient, since it is better to estimate simultaneously y_t and its conditional variance. Thus, Engle in 1982, showed that it was possible to simultaneously model the mean and the variance of a series. The proposed model is:

$$\varepsilon_t = v_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2} \quad (7)$$

Where v_t is a white noise process in which $\sigma_v^2 = 1$, y_t y ε_{t-1} They are independent of each other. In addition, the parameters from α_0 and α_1 must be constants such that $\alpha_0 > 0$ y $0 \leq \alpha_1 \leq 1$, being this last condition which ensures that the process is stable.

GARCH Model

In 1986, Bollerslev extends Engle's work by developing a technique that allows conditional variance to be an ARMA process. Being the error of said process:

$$\varepsilon_t = v_t \sqrt{h_t} \quad (8)$$

Where $\sigma_v^2 = 1$ and $h_t = E_{t-1}(\varepsilon_t)^2$. Being v_t a white noise process, its conditional mean is equal to zero. For its part, the conditional variance is $E_{t-1} \varepsilon_t^2 = h_t$, where the conditional variance of ε_t is an ARMA process given by h_t . Thus, the GARCH model:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i} \quad (9)$$

Where:

- $\sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$ they are the lags of error
- $\sum_{i=1}^p \beta_i h_{t-i}$ are the lags of the conditional variance.

So that:

- A GARCH model (1,0) has the form $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$, which is identified by the lags of the squared error ε_{t-1}^2 (Equivalent to an ARCH model (1))
- A GARCH model (0,1) has the form $h_t = \alpha_0 + \beta_1 h_{t-1}$, which is identified by the lags of the variance h_{t-1}

Another way of representing the stochastic process of the exchange rate is through lag operators, so that:

$$h_t = \alpha_0 + A(L)\varepsilon_t^2 + B(L)h_t \quad (10)$$

According to Bollerslev (1986), the GARCH process (p, q) as defined in (10) is stationary in the broad sense with $E_t(\varepsilon_t) = 0$, $\text{Var}(\varepsilon_t) = \alpha_0(1 - A(1) - B(1))^{-1}$ $\text{Cov}(\varepsilon_t, \varepsilon_s) = 0$ for $t \neq s$ only if $A(1) + B(1) < 1$. This condition of stationarity is important to ensure that the moments of the normal distribution are finite.

The benefit of a GARCH model is that ARCH models of higher order can have a more parsimonious GARCH representation, this makes it easier to identify and estimate, as well as implying less restrictions on the coefficients. Another key characteristic is that the conditional variance of the perturbations of the exchange rate sequence acts as an ARMA process.

Just as an ARMA process is identified with its simple Autocorrelation (FAC) and partial (FACP) functions of the perturbation term, the FACP of the quadratic residuals can help to identify the order of the GARCH process, for this a test of Lagrangian multiplier McLeod & Li (1983).

3. Econometric Methodology

Description of the Data

To carry out the estimation of the model, the daily nominal exchange rate of the Mexican peso against the US dollar was considered, during the period from January 1, 1995 to April 26, 2017, with a total of 8,152 observations. The data was obtained from the International Monetary Fund, from its section International Financial Statistics (IFS for its acronym in English).

Analysis of the exchange rate: Stationarity of the Series

The analysis of the time series begins with the preliminary verification of the stationary nature of the exchange rate. In the inspection of graph 3, it can be observed that the variable (peso / dollar exchange rate) is non-stationary in levels, due to its strong upward trend. In this way, in order to prove the stationarity in the logarithm of the exchange rate, the Dickey-Fuller Augmented Test (ADF) and the Phillips-Perron (PP) tests were used. See table 1.1.



Graph 3 Peso-dollar exchange rate (monthly). 1995-2017
Source: Own elaboration with results of the software R 3.3.3

EN NIVELES					
Tipo de cambio pesos/dólar	DFA		PP		Orden de integración
	t-estadístico	Probabilidad	t-estadístico	Probabilidad	
Intercepto	-0.584131	0.8718	-0.809078	0.8160	I (1)
Tendencia e intercepto	-2.141430	0.5219	-2.467433	0.3445	I (1)
Sin tendencia ni intercepto	1.950423	0.9883	1.850876	0.9851	I (1)
EN DIFERENCIAS					
	DFA		PP		Orden de integración
	t-estadístico	Probabilidad	t-estadístico	Probabilidad	
Intercepto	-40.47903	0.0000	-84.76458	0.0001	I (0)
Tendencia e intercepto	-40.47842	0.0000	-84.75945	0.0001	I (0)
Sin tendencia ni intercepto	-40.41206	0.0000	-84.71580	0.0001	I (0)

Table 1 Unit root test

Source: Own elaboration with software results from Eviews 9.0

The results obtained from the tests (ADF)¹ and (PP) in table 1.1, they show that when applying the algebraic transformation of the logarithm of the exchange rate (TC) pesos / dollar per month, in level, the variable showed that the critical values at a level of significance of 1% do not reject the null hypothesis that the logarithm of the variable in levels is non-stationary.

Therefore, to avoid the problem of spurious regression² that could arise when performing the regression of a non-stationary series of time, it is necessary to transform the series through logarithmic differences. In this sense, the unit root analysis confirmed the stationarity after taking the first differences of the logarithm of the TC peso / dollar. Defined as follows:

$$\Delta y_t = y_t - y_{t-1} = \log\left(\frac{TC}{TC_{t-1}}\right) = \log\left(1 + \frac{TC - TC_{t-1}}{TC_{t-1}}\right)$$

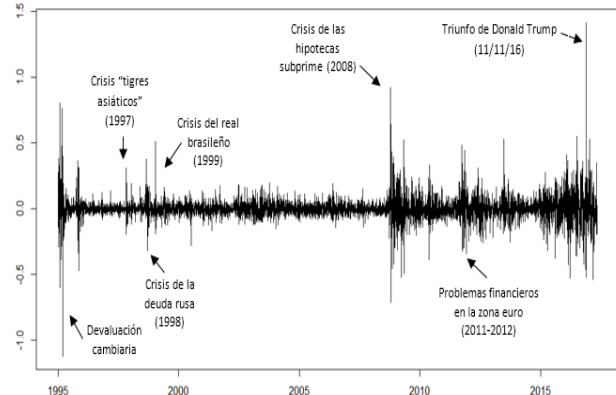
Where:

$$y_t = \log(TC)$$

However, despite the fact that the first differences in logarithms of the TC are stationary, Figure 4 suggests that during most of the period analyzed, the different economic and social events that have caused a rise or fall in the exchange rate volatility of the Mexican economy. Thus, for example, the devaluation of the Mexican peso in December 1994 is shown, which resulted in low levels of international reserves and a growing mobility in international capital flows. In this sense, there are other periods of volatility, in the years 1997, 1998 and 1999.

Which according to Caballero (2016) can be attributed to the crisis of the "Asian tigers", which occurred in mid-1997, the crisis of the Russian debt, in 1998, and the crisis of the Brazilian real, in 1999.

Also, it can be observed that the years 2005, 2006 and 2007 are periods of controlled volatility. On the other hand, the periods of high exchange volatility have their origin in the subprime mortgage crisis, which occurred in the United States in 2008 and reached its effects in 2009 and 2010, coupled with this, the exchange rate presented another period of volatility due to the worsening of the financial problems in the Euro Zone that generated pressures on the exchange rate at the end of 2011 and the beginning of 2012. Finally the historical fall of the peso before the triumph of Donald Trump in November 2016 again pressures on the exchange rate.



Graph 4 Percentage change of peso-dollar exchange rate, 1995-2017

Source: Own elaboration with software results from R 3.3.3

¹ The Dickey-Fuller (DFA) augmented test is a DickeyFuller test version for much larger and more complicated time series models. The DFA is a negative number and the more negative the statistic (with respect to critical values) the stronger the rejection of the null hypothesis about the existence of a unitary root or nonstationarity. The null hypothesis is accepted because the value of DFA is lower in absolute value (less negative) than the critical value of McKinnon at 5% (Mata, 2003, p.30 y 32).

² The problem of spurious regression, is that generally the results tend to be admitted as good, although in reality they are only due to casual aspects.

Modeling the stochastic process structure (ARIMA) (Box-Jenkins Methodology).

The objective of the Box-Jenkins methodology (also technically known as ARIMA methodology) is to identify and estimate a general class of models capable of forecasting real time series, through a statistical method that can be interpreted as a generator of information. the sample. This method consists of three stages:

1. Identification: For the choice of model, the command "auto.arima" of software R 3.3.3 was used. which automatically selects the order of the ARIMA process minimizing the Aikakie Criterion (AIC) and / or the Schwarz Bayesian Information Criterion (BIC)
2. Estimation: In this stage tentative models are adjusted and the parameters of each of the models identified in the previous phase are estimated (see table 1.2).
3. Diagnosis: This phase was carried out the tests of correct specification to ensure that the residues mimic the process of a white noise, that is, the residues of the optimal model were plotted, their simple and partial correlograms, the unit root tests were applied , and another proof of non-autocorrelation.

	Modelo 1 ARMA (5,2)	Modelo 2 ARMA (2,2)	Modelo 3 ARMA (1,1)	Modelo 4 MA (3)	Modelo 5 MA (1)
AR (1)	1.4836*** (128.3564)	1.0655*** (7.7498)	-	-	-
AR (2)	-1.0290*** (-79.1647)	-0.4106*** (-3.3250)	-0.8552*** (-13.0466)	-	-
AR (4)	0.0719*** (6.1795)	-	-	-	-
AR (5)	-0.0906*** (-9.2039)	-	-	-	-
MA (1)	-1.4718*** (-216.4884)	-1.0503*** (-7.4505)	-	0.0195* (1.7664)	-
MA (2)	0.9691*** (145.1894)	0.3495*** (2.6960)	0.8298*** (11.7548)	-0.0436*** (-3.9602)	-0.0401*** (-3.6226)
MA (3)	-	-	-	-0.0845*** (-7.6545)	-
Constante	0.0161** (2.4349)	0.0161** (2.4697)	0.0163** (2.1825)	0.0162** (2.4097)	0.0016** (2.2389)
Observaciones	8151	8151	8151	8151	8151
R ²	0.0176	0.0070	0.0023	0.0086	0.0016

*t-estadístico entre paréntesis

Nivel de significancia ***p < 0.001, ** p < 0.05, * p < 0.1

Tabla 2 Estimación de los modelos ARIMA: Metodología Box-Jenkins³

Source: Own elaboration with software results from E-views 9.0

Modeling the volatility of the exchange rate in Mexico

To identify the possible volatility existing in the time series of the Exchange Rate, a regression is made of the logarithmic difference of the variable over a constant and its perturbations are obtained from this equation. From the data, the following regression was obtained by MCO:

$$Y_t = \beta_1 + u_t \quad (11)$$

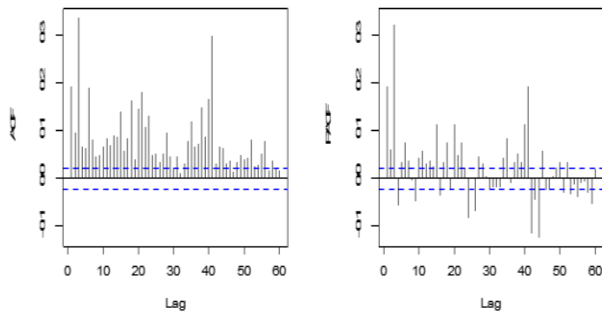
Where Y_t = percentage change in the exchange rate and u_t = random error term

$$\hat{Y}_t = 0.016300$$

$$t = 2.147921$$

³ * The scripts represent models that did not turn out to be statistically significant

From the previous regression the residuals were obtained (ϵ_t) and were squared (ϵ_t^2) to use them as a measure of volatility. Being a quadratic value, it will be high in periods in which large changes in prices are experienced and small when there are moderate changes in the prices of said goods.



Graph 5 Correlogram of the Quadratic Residues
 Source: Own elaboration with software results from R 3.3.3

To formally verify that there is an ARCH or GARCH effect in the TC, the test proposed by McLeod and Li (1983) is used, its methodology involves two steps:

Step 1. The Ordinary Least Squares (OLS) method is used to estimate the most appropriate regression equation, and denote the residuals.

Step 2: Delay the square debris on a constant and on the delay of the values “q” $\epsilon_{t-1}^2, \epsilon_{t-2}^2, \epsilon_{t-3}^2, \epsilon_{t-q}^2$, that is, an estimation of the form is made:

$$\epsilon_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 \quad (12)$$

If there are no ARCH or GARCH effects, the estimated values of α_1 a α_q must be zero. Therefore, this regression will have a little explanatory effect so that the coefficient of determination (ie, R^2 usual) It will be quite low.

Using a sample of residue T, under the null hypothesis of absence of ARCH errors, the test statistic TR^2 converges to a distribution χ^2 with q degrees of freedom.

If TR^2 is large enough, the rejection of the null hypothesis that α_1 a α_q are jointly equal to zero is equivalent to accepting the alternative hypothesis that there is an ARCH effect. On the other hand, if TR^2 is low enough, it is possible to conclude that there are no ARCH effects.

$$\epsilon_t^2 = 0.2407 + 0.1779 \epsilon_{t-1}^2 + 0.3299 \epsilon_{t-3}^2 - 0.0625 \epsilon_{t-4}^2 + 0.0318 \epsilon_{t-5}^2 \quad (13)$$

Their respective "t statistic" values are (5.24, 16.31, 31.60, -5.59 and 3.05). The nature of the model indicates that its quadratic disturbances manifest a considerable influence on the peso / dollar exchange rate. This indicates that if the volatility was high in the previous period, it will remain high in the current period, which indicates an accumulation of volatility. The sample value of the F statistic for the null hypothesis that the coefficients $\alpha_1, \alpha_3, \alpha_4$ and α_5 are equal to zero is 342.86. And since lags 1, 3, 4 and 5 are significant (all coefficients have a statistical t greater than 2), the null hypothesis is rejected that there is no ARCH error at any level of significance and the existence of an ARCH or GARCH effect to explain the volatility in the TC.

Although the estimation of an ARCH or GARCH model allows to model the volatility of the TC, it is necessary to take into account the role of the structural change in the variable, since it can alter the variance and increase the volatility of the TC. Table 1.3 and Figure 1.1 show the dates of the four break points detected in the TC for the period analyzed.⁴

⁴ The procedure used to determine the breakpoints in the series was developed using the "breakpoints" command in the "strucchange" library developed by Zeileis (2015).

First break point	06/05/1998
Second break point	05/12/2002
Third break point	08/10/2008
Fourth break point	21/12/2013

Table 1.3 Breakpoints in the exchange rate (1993-2017)
 Source: Own elaboration with information obtained from the software R 3.3.3

With these results, a dummy variable is constructed to correct the effect of the structural changes on the TC and verify if this change has had significant effects on the parameters in the period analyzed. This fact is shown in equation 14, where the dummy variable that reflects the structural changes was statistically significant at a confidence level of 5%. In this sense, with the introduction of a dichotomous variable changes were detected by assigning the value 0 before and after the structural change, and 1 when exactly a breakpoint occurs in the variance of the series.

$$\varepsilon_t^2 = 0.6156 + 0.1772 \varepsilon_{t-1}^2 + 0.3292 \varepsilon_{t-3}^2 - 0.0629 \varepsilon_{t-4}^2 + 0.0311 \varepsilon_{t-5}^2 - 0.0328 \text{ dummy} \quad (14)$$

Their respective statistical values are: 3.37, 16.24, 31.52, -5.63, 2.98 and -2.12, respectively. Once the structural breaks in the variance are considered and by accepting the existence of an ARCH or GARCH effect to explain the volatility in the TC, the empirical strategy consists of estimating different models to measure the adjustment of the same: the alternatives include models that follow an autoregressive process of order $\rho = 1,2$ being the order of the process of moving averages; $k = 1,2$, and the ARCH terms in the variance modeling (see table 1.5).

For the selection of the best model, two traditional information criteria are used: the Akaike information criterion (1976), and the Bayesian information criterion of Schwarz (1978)⁵. The estimate was made jointly for the equations of mean and variance, in such a way that the order of the autoregressive processes in each responds to global selection criteria. The results are presented in table 1.4.

	Modelo 1 ARMA (1,2)	Modelo 2 ARMA (2,2)	Modelo 3 MA (2)	Modelo 4 MA (1)	Modelo 5 MA (2)
AR (1)	0.7150 (24.4723)	-	-	-	-
AR (2)	-	-0.2397 (-2.3359)	-	-	-
MA (1)	-0.6738 (-22.1051)	0.0467 (5.0925)	0.0562 (5.0713)	0.0585 (5.3031)	-
MA (2)	-0.0763 (-9.6190)	0.1971 (1.8694)	-0.0304 (-2.5575)	-	-0.0314 (-2.6326)
Ecuación de la Varianza					
Constante	0.2004 (45.3405)	0.2006 (52.5983)	0.0035 (19.9510)	0.0035 (19.8513)	0.0035 (20.3808)
ε_t^2	0.0451 (28.6046)	0.0454 (28.5800)	0.0838 (46.5701)	0.0840 (46.6093)	0.0843 (46.4599)
GARCH (1)	0.4951 (49.6912)	0.4954 (57.1514)	0.9089 (537.8678)	0.9086 (534.3986)	0.9084 (539.2924)
Observaciones	8150	8149	8151	8151	8151
R ²	0.0094	0.0025	-0.0002	-0.0012	0.0009
Criterio Akaike	1.7921	1.7920	1.3352	1.3357	1.3375
Criterio Schwarz	1.7973	1.7972	1.3391	1.3392	1.3410

Table 1.4 Estimated models for exchange volatility
 Source: Own elaboration with information obtained from the software E-views 9.0

Following these information criteria, the results indicate that of the five proposed models, the model that best adjusted to the monthly information was Model 3 - which follows a process of moving averages of order two MA (2) -, since this presented the smallest values of the statistical criteria. Furthermore, it turned out to be consistent with the structural changes, since the dummy variable that represents the breakpoints is statistically significant (table 1.5).

⁵ AIC = T * log(SRC) + 2 * n
 BIC = T * log(SRC) + n * log(T)

Where n = number of estimated parameters (p + q + constant term) and T = number of observations used.

Variable	Coefficient	Standard Error	Z-Statistic	Probability
MA (1)	0.0569	0.0111	5.112	0.0000
MA (2)	-0.0309	0.0119	-2.5946	0.0095
Equation of Variance				
C	0.0036	0.0001	19.8387	0.0000
RESID(-1) ²	0.0825	0.0017	45.9527	0.0000
GARCH(-1)	0.9091	0.0016	536.0606	0.0000
DUMMY	0.2722	0.0335	8.1250	0.0000

Table 1.5 Estimation of the ARIMA-GARCH Model
Own:Elaboration with the software Eviews 9.0

It is convenient to point out that in particular the use of the GARCH model (1,1) has specific advantages to forecast the exchange volatility, among the main ones it has that: i) it allows more parsimonious specifications than the ARCH model, avoiding the over parameterization of the specification and, thereby, facilitating the maintenance of the model; ii) among the ARCH / GARCH family models is the most common specification, which is why its properties are already known; and iii) it fits very well the data of financial variables, mainly when the observations are of high frequency (Ortiz, et. al, 2012).

Results

Financial time series, such as the exchange rate, often present the phenomenon of volatility accumulation, that is, there are periods in which their prices show high variations and others in which they present relative stability. In this sense, several models have been developed in the empirical literature to estimate the volatility of financial variables.

The models most commonly applied to estimate the volatility of the exchange rate are the autoregressive models with conditional heteroscedasticity (type ARCH and GARCH). In this regard, the objective was to analyze and explore through the ARCH and / or GARCH models the evolution of volatility in the exchange rate in Mexico.

In the previous section, the estimation of the volatility of the exchange rate in Mexico for 1995-2017 is developed using these models of conditional heteroscedasticity. From the above results, expressed in equation (13), the existence of ARCH and GARCH effects can be clearly observed. It is found that the volatility of the exchange rate in Mexico is an example of an ARIMA (0,0,2) + GARCH (1,1) model in which it is proven that the conditional volatility in the exchange rate in Mexico has been highly persistent. As such, for the Mexican economy it must be anticipated that any shock that generates uncertainty in the foreign exchange market will tend to show little tendency to dissipate.

Conclusions

It is important to monitor the dynamics of the exchange rate, whether in developed or developing countries, since it is an important transmission channel of monetary policy. Their behavior not only has an impact on the price level, but also on the productivity and performance of financial instruments. This is why it becomes necessary to have models that explain not only the reactions to yields and risks, but also regarding volatility.

In the case of Mexico, the management of the exchange rate has been of vital importance, which despite being classified as a free floating regime, has been characterized by constant interventions by the monetary authorities in the exchange market to maintain a certain value of the currency, thus avoiding strong depreciation, becoming the main tool to achieve the objective of stability of inflation. In this way, the analysis and results achieved in this document show that, the exchange rate presents volatility, when presenting conditional heteroscedasticity in its residuals, which was verified through the ARCH and GARCH models. However, knowing the existence or not of volatility becomes insufficient if the factors that have triggered it are not known.

In the case of Mexico, this has been generated not only by internal shocks, but also by international events, and in the last decade, those originating from the environment of the US economy (subprime crisis and election of Donald Trump) have been those of greater importance, due to the high economic dependence that this country has.

Added to this, this volatility has a persistent effect, so that oscillations in previous periods show a high degree of persistence. Knowing this behavior is important to make correct predictions about future values, ignoring volatility, can lead to errors in the estimates, which could have a negative effect on investors and risk managers.

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