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Geometric analysis as a design base for implementing joints assisted for the displacement of components in tilting furnaces

Análisis geométrico como base de diseño para la implementación de articulaciones asistidas para el desplazamiento de componentes en hornos basculantes

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Abstract

Various devices in productive chains implement parts that move through pneumatic, hydraulic, electrical systems, or some combination. Articulation-assisted displacement systems implemented by hydraulic systems are of particular interest, specifically, those used to lower bodies concerning support points with a structure of degrees of freedom of movement. An application in the metallurgical industry establish since processes at high temperatures and elements of great mass make human manipulation impossible. However, introducing these automation operations makes standardizing and controlling processing times possible. In this work, the procedure for the mathematical approach through principles of analytical geometry and trigonometry, to be develop as a contribution to the domain of knowledge for the design of articulated systems. The above applied to the displacement requirements in a melting furnace of metal alloys allowed selection of the characteristics of hydraulic components to implement in the consolidation of industrial equipment.

Angle Joint Design, Hydraulic Systems, Assisted Displacement

Resumen

Diversos dispositivos en cadenas productivas implementan partes que son desplazadas mediante sistemas neumáticos, hidráulicos, eléctricos o alguna combinación de éstos. Los sistemas de desplazamiento asistidos por articulaciones en las cuales se implementan sistemas hidráulicos son de interés particular. Específicamente, los que se utilizan para abatir cuerpos respecto a puntos de apoyo que cuenten con una estructura de grados de libertad de movimiento. Una aplicación es en la industria metalúrgica ya que existen procesos a elevada temperatura y elementos de gran masa que imposibilitan una manipulación humana. No obstante, la introducción de estas operaciones de automatización permite estandarizar y controlar los tiempos de procesamiento. En este trabajo se desarrollará el procedimiento para el planteamiento matemático, a través de principios de geometría analítica y trigonometría, como aporte al dominio del conocimiento para el diseño de sistemas articulados. Lo anterior aplicado a los requerimientos de desplazamientos en un horno de fusión de aleaciones metálicas correspondientes, permitió seleccionar las características de componentes hidráulicos para ser implementados en la consolidación de un equipo industrial.

Diseño de Articulaciones Angulares, Sistemas Hidráulicos, Desplazamiento Asistido

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December 2023, Vol.7 No.13 7-14

Introduction

There are various industrial furnace products for carrying out activities in high-temperature conditions. Metal alloy melting furnaces are an example of these devices (Nabertherm, 2023). These pyrometallurgical activities are essential to manufacturing processes (Mullinger & Jenkins, 2008). Therefore, there is relative interest in the design of assistance for moving rigid elements with a large mass. Hydraulic cylinder systems are ideal, thanks to their high load capacity, (DeKun, Yuan, SiYuan, Peng, & NianLi, 2023), (Drew, Edge, Barker, Darling, & Owen, 2005), (Janebrink, 2014), (Kwasi-Adzimah, Samuel-Akinwonmi, & Sebbeh, 2012).

In the case of furnaces, various displacement functions assisted by power systems implement rigid, articulated elements to perform controlled displacements. **Figures 1(a) and 1(b)** show images from different perspectives of non-ferrous metal fusion equipment used in the metallurgical industry to manufacture parts or pieces.

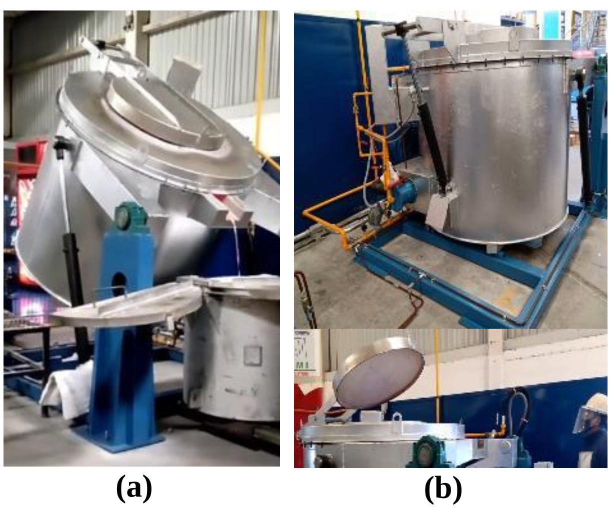


Figure 1 Melting furnace implementing: a) joint for tilting, b) joint for opening the lid
Spurce: Images property of Mechatronics DDMI COMBUSTION. https://www.youtube.com/watch?v=LP3Wmo8rSlU

As can be seen, the hydraulic cylinders assist, with a possible speed adjustment, in the two main events: 1) the movement in the joint for opening the lid and 2) the tilting of the furnace, therefore, the determination of its adaptation according to the characteristics of the extension and retraction lengths of the commercialized elements.

This work analyzes the trajectories with geometric lines of the structural joints for a prototype of a non-ferrous metal smelting furnace. The objective of specifying the cylinders for each displacement function required the mathematical formulation of the load balances.

Methodology

In the methodology adopted, an analysis of the geometry of assisted joints implemented in collapsible melting furnaces allowed for establishing a design basis. In particular, without establishing an exclusive criterion for different similar devices, the following specific objectives were set:

- Mathematically pose the geometric problem considering the degrees of freedom of movement at the assembly points.

- Use a physical model's dimensions to verify the previous point's mathematical formulation and graphically show the displacement trajectories.

Figure 2 complements the visual information of **Figure 1(b)** with the schematization of the joint structure used to fold down the furnace lid.

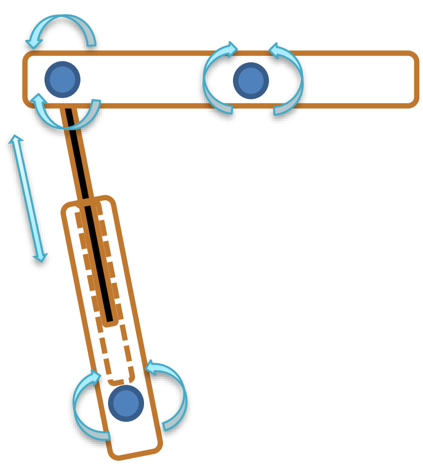
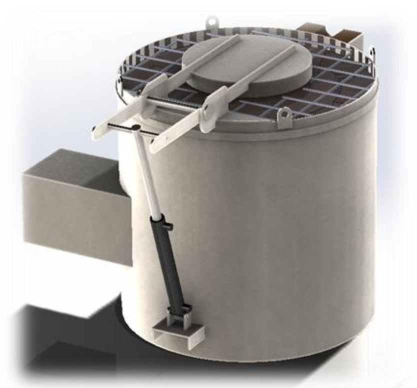


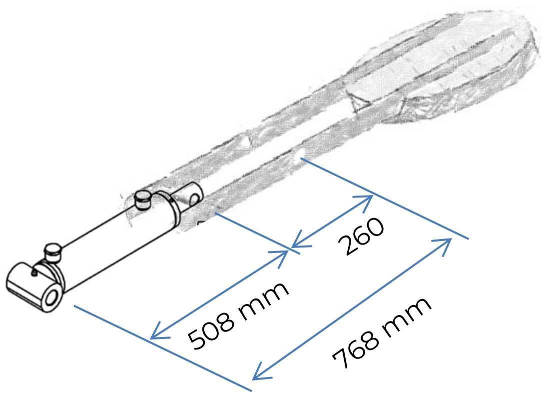
Figure 2 Schematization of elements in the joint of the mechanism for opening the furnace lid.
Source: Own elaboration in Microsoft PowerPoint 2016

Article Journal-Mathematical and Quantitative Methods
December 2023, Vol.7 No.13 7-14

The development of the design basis for the manufacture of each joint was born from an analysis of plane geometry and trigonometry. Therefore, each mathematical approach to the problems considered the degrees of freedom of movement at the assembly points (hinges). In the case of the lid, the dimensions of the physical model presented in **Figure 3** are the reference for the calculations. The results obtained from the displacement trajectories showed certainty once they were analyzed graphically.



(a)



(b)

Figure 3 Specifications: a) of the CAD model of the furnace and b) of the hydraulic cylinder-lid arrangement used for the geometric analysis of the corresponding joint
Source: Own creation in SolidWorks 2018.

The specification of available cylinders was obtained from the Table in **Figure 4** from a commercial supplier (Magister_Hydraulics, 2015-2023). **Table 1** shows further specifications of the retraction and expansion length data.

2" BORE		Welded Double Acting Hydraulic Cylinders Swivel Eye 3500 PSI								
SKU	BORE (INCH)	STROKE (INCH)	ROD Ø (INCH)	RETRACTED (INCHES)	EXTENDED (INCHES)	PIN Ø (INCH)	PORT SIZE	COLUMN LOAD	WEIGHT LBS	LIST PRICE
WSE 2x6	2	6	1.25	16	22	1	SAE 6	9,400	16	$107
WSE 2x8	2	8	1.25	18	26	1	SAE 6	9,400	17	$111
WSE 2x10	2	10	1.25	20	30	1	SAE 6	9,400	18	$115
WSE 2x12	2	12	1.25	22	34	1	SAE 6	9,400	19	$121
WSE 2x14	2	14	1.25	24	38	1	SAE 6	9,400	21	$126
WSE 2x16	2	16	1.25	26	42	1	SAE 6	9,400	22	$128
WSE 2x18	2	18	1.25	28	46	1	SAE 6	9,400	24	$130
WSE 2x20	2	20	1.25	30	50	1	SAE 6	9,400	26	$133
WSE 2x24	2	24	1.25	34	58	1	SAE 6	8,400	28	$139

Figure 4 Specifications of hydraulic cylinders sold
Source: (Magister_Hydraulics, 2015-2023)

As specified in **Figure 3(b)**, a 508 mm (20 in) retracted cylinder coupled to a 260 mm straight section determines the position of the ball joint for the furnace lid opening and closing mechanism. **Figure 5** presents the geometric scheme of the joint in the closed lid position.

R, in	E, in	R, mm	E, mm
16	22	406	559
18	26	457	660
20	32	508	813
22	34	559	864
24	38	610	965
26	42	660	1067
28	46	711	1168
30	50	762	1270
34	58	864	1473

Table 1 Lengths of retracted and extended cylinders according to commercial models

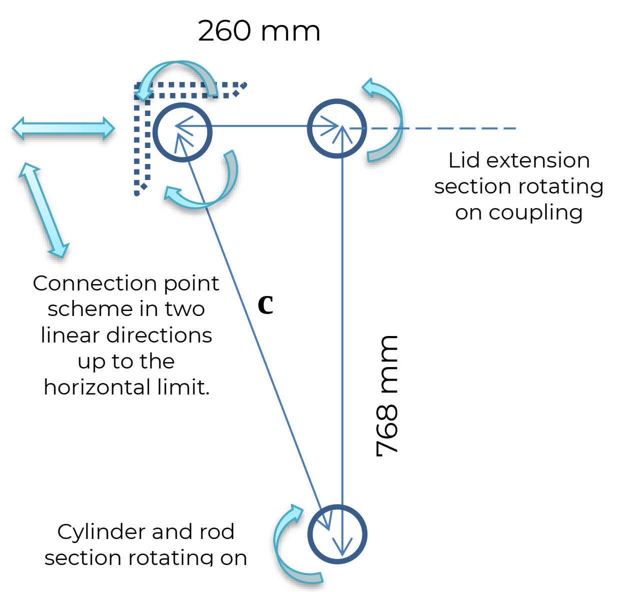


Figure 5 Geometric diagram of articulation in closed lid position with an extension of 260 mm
Source: Own creation in Microsoft PowerPoint 2016

Equation 1, defined by the Pythagorean Theorem, allowed us to calculate the length **c**. The result defines the maximum extension required in the cylinder.

$$\boldsymbol{c} = \sqrt{(260)^2 + (768)^2} = 811\ mm \quad \textbf{(1)}$$

The calculated magnitude of 811 mm is in the extension interval of the WSE 2x10 cylinder indicated in the Table of **Figure 4**, which has an extension limit of 812.8 mm according to the data reported in **Table 1**.

When the lid is in the path between the closed and fully opened position, the analysis of the geometric system changes to the scheme shown in **Figure 6**. In this new scheme, two right-angled triangles appear coinciding in the horizontal partition defined as **z**, representing the variable separation from the vertical concerning the position of the R_2 kneecap. The magnitudes of the distances **x** and **y** change because of the arc drawn in the lid's path due to the cylinder's retraction or extension determined by the magnitude **a**.

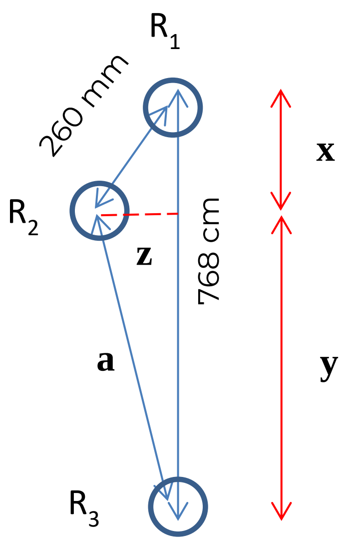


Figure 6. Diagram of the joint in an intermediate process of closing and total opening
Source: Own creation in Microsoft PowerPoint 2016.

The scheme also aims to define that the value of the length **x** is between 0 mm and 260 mm, while the length **y** is in an interval of 768 mm to 508 mm. Note that the sum of both magnitudes is constant (768 mm). In this way, based on the structure of **Figure 6**, **Equations 2 and 3** are defined, which, when equalized and developed, determine the relationship between the lengths **y** and **a** through **Equation 4**.

$$\mathbf{z}^2 = 260^2 - \mathbf{x}^2 \qquad \textbf{(2)}$$

$$\mathbf{z}^2 = \mathbf{a}^2 - \mathbf{y}^2 \qquad \textbf{(3)}$$

$$\mathbf{y} = \frac{\mathbf{a}^2 + 768^2 - 260^2}{2(768)} \qquad \textbf{(4)}$$

The corresponding clearances also lead to calculating the magnitudes **z** and **x**. Afterwards, the information generated determines the calculation of the angle ratio of the upper right triangle. In particular, the angle **β** is directly related to the degree of collapse of the lid, as shown in **Figure 7** in the geometric structure. As a result, the displacement relationship added to a moment at the support points will generate a slight deviation from a presumably linear relationship to represent the movement of the joint. However, the polynomial relationship of **a** concerning **x** and **y** is of second order.

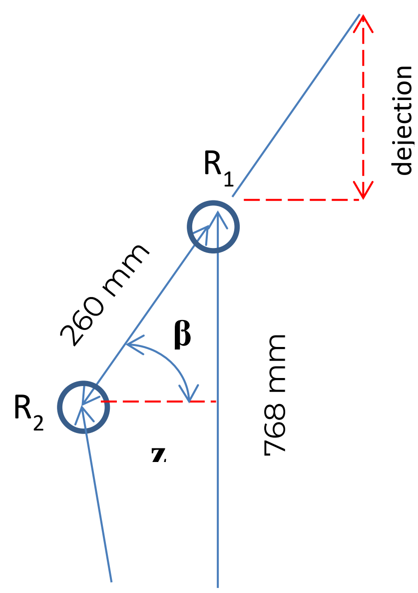


Figure 7 Diagram of the structure of the articulated lid defining the angle **β** with respect to the horizontal for its folding
Source: Own creation in Microsoft PowerPoint 2016.

The second joint required for the model furnace has a different movement relationship because displacement starts from a structure similar to a porch with vertical pillars and a horizontal beam. **Figure 8** presents the dimensions of the relevant sections.

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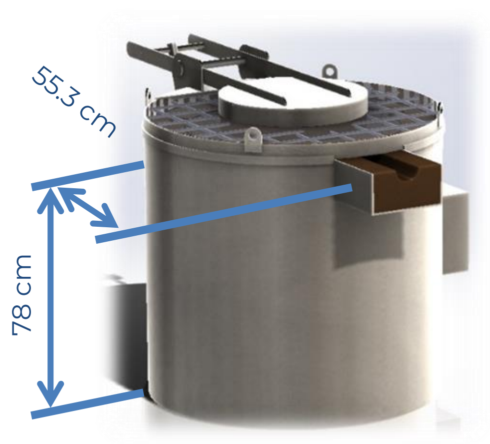

Figure 8 Furnace model for the implementation of the tilting system on each flank
Source: Own creation in SolidWorks® 2018.

According to **Table 1**, the best coupling for the joint corresponds to the cylinder with a length of 864 mm in the retracted state. The total tilt of the furnace, that is, the ability to rotate it 90° from the horizontal, must be developed during cylinder extension. A right triangle drawn in this last position demonstrates that the magnitude of the extended cylinder **c** is determined again with the Pythagorean Theorem expressed by **Equation 5**. **Figure 9** shows the implementation strategy of the cylinder.

$$\boldsymbol{c} = \sqrt{(133.3)^2 + (55.3)^2} = 144.3\ cm \qquad (5)$$

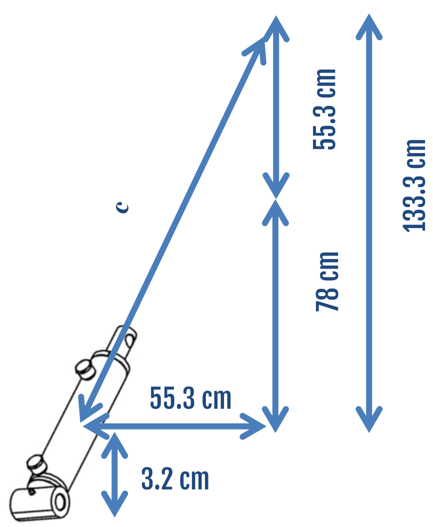

Figure 9 Diagram of the joint in the extended cylinder state for each flank of the furnace
Source: Own creation in Microsoft PowerPoint 2016.

Additionally, **Figure 10** represents the joint structure's geometric approach during the furnace's tilting. **Equations 6 and 7** emerged through the analysis developed for this system.

$$\boldsymbol{a}^2 = \boldsymbol{x}^2 + \boldsymbol{y}^2 \qquad \textbf{(6)}$$

$$L^2 = (\boldsymbol{y} - H)^2 + (L - \boldsymbol{x})^2 \qquad \textbf{(7)}$$

a represents the length of the extended cylinder or hypotenuse, in any scheme during the tilting of the furnace, associated with the right triangle that also defines the angle θ.

Equation 7, in a complementary manner, describes the relationship of distances formed by the right triangle formed by the displacement of the rigid section coupled to the furnace from its horizontal position until it reaches the vertical position. Obtaining **Equation 8** corresponds to developing **Equation 7** and substituting the solution for **x** from **Equation 6**.

$$y^2(4L^2 + 4H^2) - y(4H^3 + 4a^2H) + (2a^2H^2 - 4a^2L^2 + H^4 + a^4) = 0 \qquad \textbf{(8)}$$

The magnitudes of L and H can vary for different furnace dimensions; however, for the specific case under study, they have a magnitude of 55.3 cm and 78 cm, respectively, according to **Figure 9**.

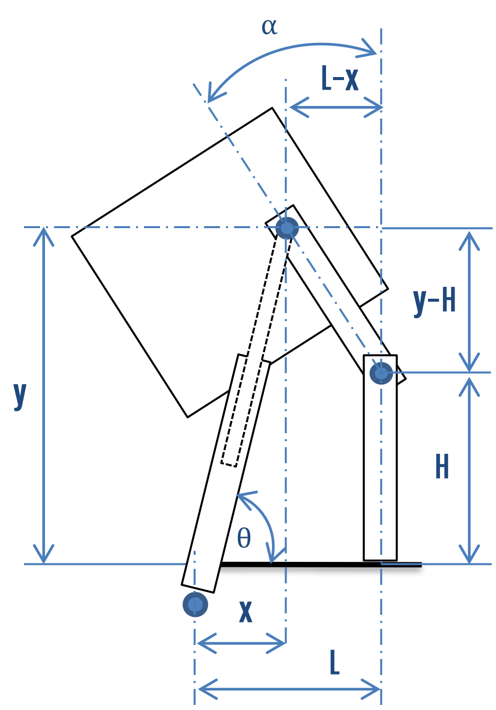

Figure 10 Geometric analysis of the joints for furnace tilting
Source: Own creation in Microsoft PowerPoint 2016.

Results

The analysis of the curves in **Graph 1** allows us to understand the variation of the lengths of **x**, **y**, and **z**, defined by **Equations 2 to 4**, for the opening hinge mechanism of the furnace lid.

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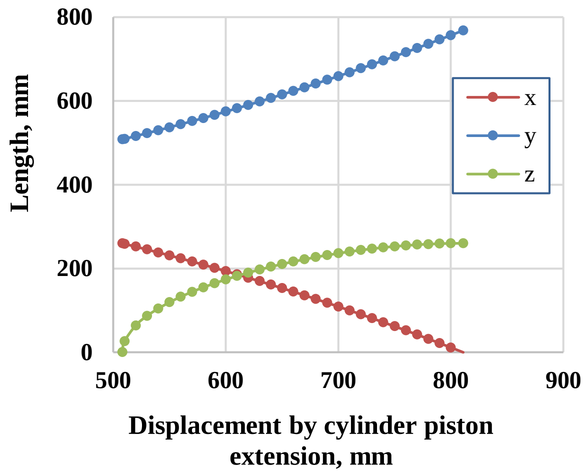
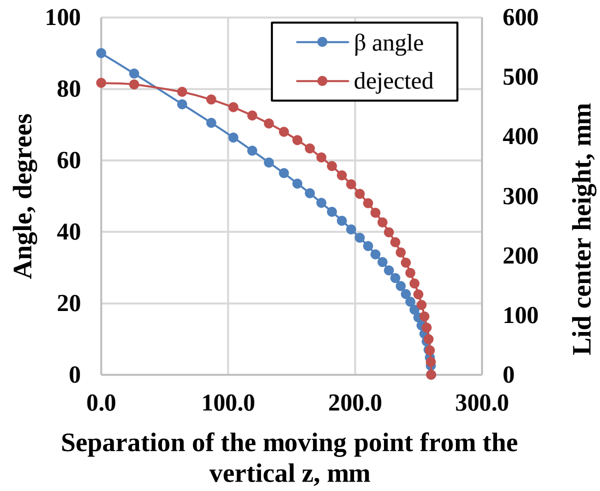


Graphic 1 Solution curves of **Equations 2 to 4** of the articulated system for opening the furnace lid according to **Figure 5**
Source: Own creation in Microsoft Excel 2016.

The trend of the magnitudes of the displacements for **x** and **y** are opposite, reaching the maximum and minimum length, respectively, in the fully open lid event (curves with blue and brown filled lines and circles). The opposite case occurs in the fully closed lid event where the length of **x** is zero.

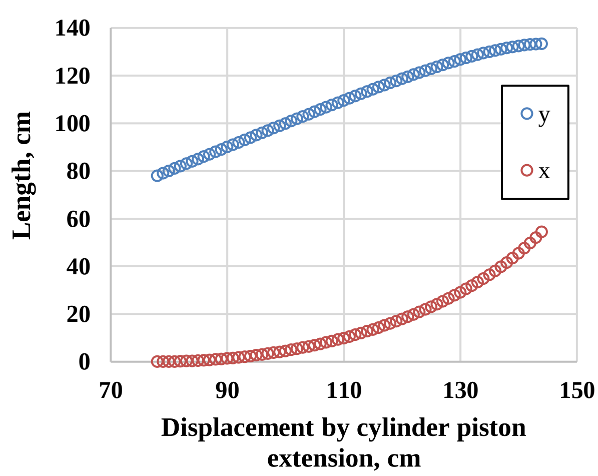
Note that the most significant parabolic behavior corresponds to the definition of the length **z** (curve with gray-filled lines and circles), which is related to the displacement of the patella R2 simultaneously in the vertical and horizontal directions.

On the other hand, the analysis of the trajectory of the lid during the collapse in the closing event determines that it falls rapidly, losing the angle concerning the vertical (curves with blue and brown lines with filled circles, respectively). The length of the stroke and the advance angle decreased as the cover reached the horizontal position according to the outline of the curves in **Graph 2**.



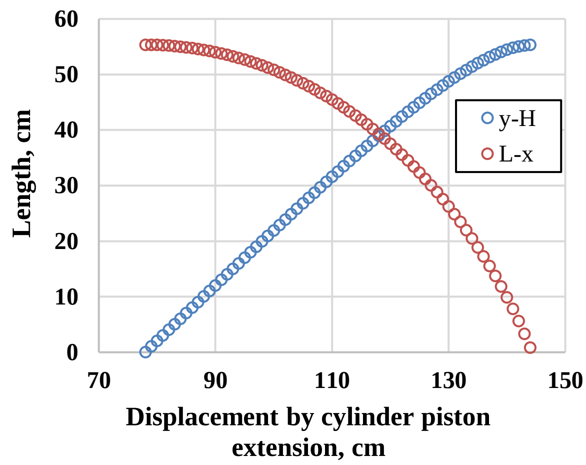
Graphic Curves of the evolution of the beta angle are defined in **Figure 6,** and the respective lowering of the furnace lid is according to **Figure 5**
Source: Own creation in Microsoft Excel 2016.

When tilting the furnace to empty the contents of the internal container or crucible, the curves in **Graph 3** correspond to **Equations 6 and 7** results, which define the trajectory on the vertical (**y**) and horizontal axes (**x**) of the coupling of the hydraulic cylinder with the furnace. Due to the effect of the scale, the displacement y (curve with empty blue circles) appears to have significantly linear behavior; however, by definition of **Equation 6**, the curvature in an interval around the maximum extension of the cylinder piston indicates that before From the 90° emptying position, the most relevant displacement takes place in the horizontal direction. The above is congruent with the behavior of the corresponding curve of **x** (curve with empty brown circles).



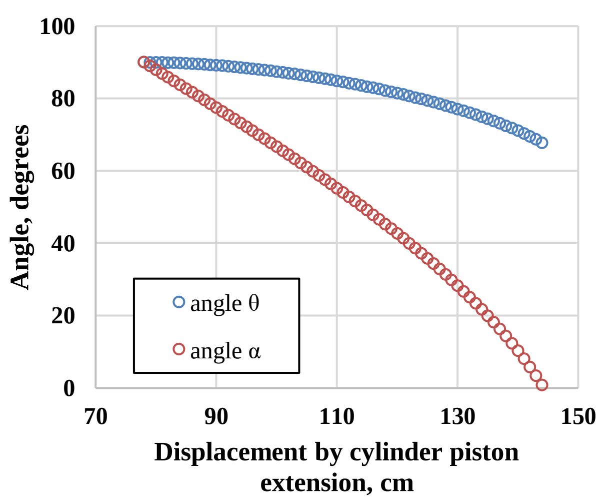
Graphic 3 Solution curves of **Equations 6 and 8** of the articulated system for tilting the furnace according to **Figure 10**
Source: Own creation in Microsoft Excel 2016.

As defined by the dimension of the diagram in **Figure 10**, the lengths specified as (y-H) and (L-x) have opposite magnitudes; that is, while one dimension is at the maximum, the other reaches a minimum of zero, as shown in the plot of the curves in **Graph 4** (blue and brown circles, respectively).



Graphic 4 Solution curves of **Equations 7 and 8** of the articulated system for tilting the furnace according to **Figure 10**
Source: Own creation in Microsoft Excel 2016.

The trends in the curves of **Graphs 3 and 4** correspond to the result of the solution of **Equation 8**, applying the solution method for a quadratic equation. In this way, it was also possible, through the translated angles technique, to determine the importance of the evolution of the angles θ and α for the tilting of the furnace, as shown with the corresponding curves in **Graph 5** (circles of blue and brown, respectively).



Graphic 5 Curves of the evolution of the angles α and θ defined in the scheme of **Figure 10**
Spurce: *Own creation in Microsoft Excel 2016.*

As can be deduced from the tilting of the furnace, the cylinder inclination angle θ only decreases from 90° to 68° (curve with empty blue circles). Not so for the angle α, which must evolve between 0° and 90° (curve with open brown circles).

Thanks

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Conclusions

The analysis of displacements in the joints' elements allows for determining the cylinders' specifications that assist the fundamental movements, whether to lower a gate or perform a tilt, as in the case of the non-ferrous metal melting furnace prototype presented. n particular, with an approach of plane geometry and trigonometry tools, it was possible to determine an adequate approximation of the trajectory of the components of both independent systems. In this regard, a change in analysis strategy is emphasized depending on the structure of the joint and its position in a superimposed Cartesian plane. In this way, determining the load capacity in the rigid elements could be analyzed to establish the degree of force supported in each position during displacement.

Additionally, given the existence of similar systems applied to various industrial processing devices, the present development can be implemented in a computer application in which it would be possible to feed the magnitudes of the target cylinder and the rigid sections to establish a valid design basis.

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