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Hilbert transform for modulation of interference fringes of liquid substances

Transformada Hilbert para demodulación de franjas de interferencia de sustancias líquidas

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Abstract

The demodulation of interferometric signals presents a challenge in the study of object, especially when working in environments with disturbances, instability or noise. This document presents a procedure to demodulate a pair of highly noisy interferograms obtained with a fiber optic Mach Zehnder interferometer, the procedure is based on using the reconfiguration of the interference fringes as a result of Hilbert Transform. The method shown is fast, it does not present susceptibilities at high frequencies, it avoids demodulating interference patterns of closed fringes and with little carrier signal, in addition, it allows to verify the mathematical equivalence of the spectral analysis using Fourier Transform. As an application model, samples of liquid substances were used, where it was possible to know changes between substances according to their density.

Hilbert Transform, Interferometry, Circular fringes

Resumen

La demodulación de señales interferométricas presenta un reto en el estudio de los cambios de objetos, sobre todo cuando se trabaja en entornos con perturbaciones, inestabilidades o ruido. Este documento presenta un procedimiento para demodular un par de interferogramas altamente ruidosos obtenidos con un interferómetro Mach Zehnder a base de fibra óptica, el procedimiento se basa en utilizar la reconfiguración de las franjas de interferencia como resultado de un análisis Hilbert. El método mostrado es rápido, no presenta susceptibilidades en altas frecuencias, evita demodular patrones de interferencia de franjas cerradas y con poca señal portadora, además, permite comprobar la equivalencia matemática del análisis espectral utilizando Transformada de Fourier. Como modelo de aplicación se utilizaron muestras de sustancias liquidas, donde fue factible conocer cambios entre sustancias de acuerdo con su densidad.

Transformada Hilbert, Interferometría, Franjas circulares

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Introduction

Several studies have been carried out to determine the physical properties of substances, which have relevance in the pharmaceutical industry, medicine, biology, engineering, among other areas, the advances in this study show a relationship between density and refractive index of the mixtures analysed through the physical and chemical characteristics of the substance, such as refractive indices, pressure, temperature, molar refraction and polarisability constant [1,2]. When the substance interacts with light, these parameters determine the change in optical path, thus developing techniques to determine changes in the properties of substances; This has given rise to non-invasive measurement methods, including expressions to determine the relationship between the density of the substance and the propagation of light in a medium, Newton-Laplace, Gladstone-Dale and Lorenz-Lorentz [3], to the integration of systems capable of detecting small displacements to determine refractive indices of gases [4], fibre optic sensors without physical contact with the material, [5], the use of interferometric refractometers [6-7], interferometers for temperature change and sample concentration [8-10].

However, the instruments and methodology reported so far require high stability and are also limited by the experimental set-up [3-4], this problem has been solved with the use of interferometers that provide the stability and measurement range that is not achieved with conventional refractometers and holographic techniques, among them the Jamin, Mach Zehnder and Michelson interferometers [4,10-11]. Despite the reliability that interferometers provide to the measurement, the problem of a later demodulation of the fringe pattern obtained arises, this pattern shows the changes before and after the deformation to later obtain the optical phase, commonly, by means of techniques based on Fourier Transform [12], Wavelet [13-18] and Hilbert [19-23], the problem increases if the interferograms obtained have a circular configuration, their carrier frequency is very low, and they also present several reflections caused by the arrangement of the instruments and the analysed material, requiring more computational work and the implementation of new spectral and temporal analysis techniques [24-32].

Although Hilbert analysis applied to closed fringes has already been studied [24], in this work we present the analysis of the difference of the reconstructed analytical signal, computationally redistribute the interference fringes in order to know the density change between substances and check the spectral dependence between the Fourier Transform and the Hilbert.

The method used is able to demodulate noisy interferograms, without carrier frequency and with high reflections, using liquid substances and with low stability.

Experimental method

To obtain the fringe pattern, a Mach Zehnder interferometer based on single-mode fibre optics, a coherent light source at 632.8 nm, a 20x microscope objective, Figure 1, was used. Where one end of the interferometer was taken as a reference, the other end is considered as the deflection which is incident on the diffusion cell, with a size of 25 cm3; both directed towards a beam splitter. The images were obtained using an XC-77 CCD camera with a pixel size of 11 × 13 μm and a computer to process the data.

For substance change analysis, water was used as the reference liquid and ethanol as the immersion liquid with 96% purity; ethanol was added into the water sample, changing its density between 0.9957 gr/cm3 and 0.8054 gr/cm3, for water and ethanol [1,33], respectively, Table 1.

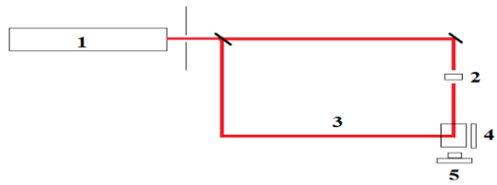

Figure 1 Mach Zehnder interferometer to obtain interference fringes of liquid substances (water-ethanol) with different density (1) He-Ne laser, (2) sample of liquid substance, (3) reference, (4) mirror, (5) CCD
Source: Own elaboration

Substance	water (ρ_1)	water-ethanol 5ml (ρ_2)	water-ethanol 10ml (ρ_3)	water-ethanol 15ml (ρ_4)	water-ethanol 20ml (ρ_5)	water-ethanol 25ml (ρ_6)	Ethanol (ρ_7)
Density (ρ) (gr/cm^3)	0.9957	0.9858	0.977	0.9689	0.9616	0.9549	0.8054

Table 1 Density of liquid mixtures
Source: [1,33]

Theoretical model and experimentation

After obtaining the interferograms with the experimental arrangement of Figure 1, they can be mathematically expressed as:

$$I_n(r,t) = a(r,t) + b(r,t)\,cos\big(\phi(r,t)\big). \quad (1)$$

Where a(r,t) is the interferogram background illumination, b(r,t) is the modulation amplitude and □(r,t) is the interference term. Traditionally, the method to know the phase difference between two interferograms makes use of the direct and inverse Fourier Transform, filters, with a separation of the interferogram, Figure 2, to calculate the phase the *arctan* function is used, so the phase difference can be determined as:

$$\Delta\varphi = \cos\left(\phi_{wx_1} - \phi_{wx_0}\right) + \cos\left(\phi_{wy_1} - \phi_{wy_0}\right). \quad (2)$$

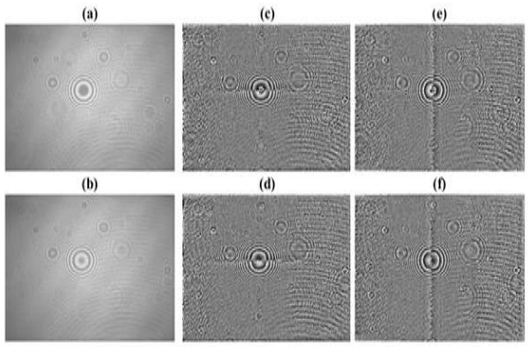

Figure 2. Optical phase of two interferograms (a) water interferogram, (b) ethanol interferogram, (c),(d) x-enveloped phase, for (a), (b), and (e),(f) y-enveloped phase, for (a), (b)
Source: own elaboration.

A normalised interferogram is achieved by eliminating the background and approximating the modulation to one, equation (3). Figure 3 illustrates the procedure to determine the phase difference Δφ using the Hilbert Transform, between two highly noisy interferograms, a bandpass filter ρ(x,y) is used first, in our case, the size that presented the smallest rms error is the full-field one. (a), (e) represent the normalised interferograms, (b), (f) the direct Fourier transform of each interferogram, (c), (g) the inverse transform of each interferogram and (d), (h) the absolute value of the Hilbert transform of the interferograms (a), (e), (i), (ii), (iii), (iv), (v), (vi), (vii), (viii), (viii), (viii), (viii), (viii), (viii) and (viii).

$$I_n(r,t) = b(r,t)\cos\big(\phi(r,t)\big). \quad (3)$$

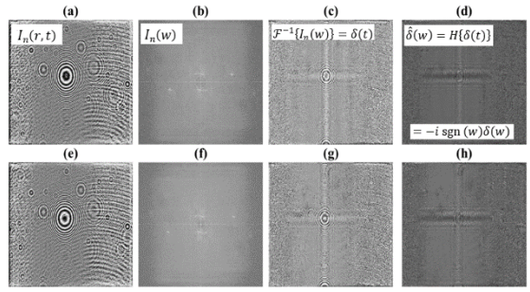

Figure 3 Process for analytical signal with Hilbert transform (a) standardised water interferogram, (e) standardised ethanol interferogram, (b), (f) Fourier transform of water and ethanol interferograms, and (c), (g) inverse Fourier transform of water and ethanol interferograms, (d) and (h) Hilbert transform of (c), (g)
Source: own elaboration.

To reconstruct the analytical signal of each interferogram we take the signal δ(t) and the absolute value of δ'(t), equation (4), Figure 4 (a), (b) represent the water and ethanol interferograms, while (c), (d) the enveloped phase for (a), (b), equation (7).

In order to determine the difference between the substances, the equation (7).

$$\mathcal{F}^{-1}\{\hat{\delta}\,(w)\} = \delta'(t) \quad (4)$$

$$\varphi(r,t) = \delta(t) + |\delta'(t)| \quad (5)$$

$$\varphi_w(r,t) = \tan^{-1}\left\{\frac{Im\{\hat{\delta}\,(w)\}}{Re\{\delta(t)\}}\right\} \quad (6)$$

$$\Delta\varphi(r,t) = \varphi_{w_A}(r,t) - \varphi_{w_B}(r,t) \quad (7)$$

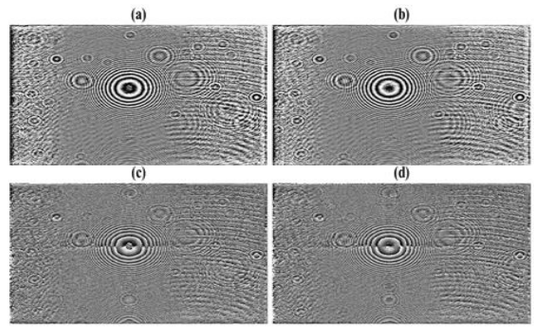

Figure 4 (a), (b) Reconstructed signal using Hilbert Transform of water and ethanol and (c), (d) Enveloped phases of (a), (b)
Source: own elaboration.

To determine the spectral dependence between Fourier Transform and Hilbert Transform, equations (2) and (5) were taken, Figure 5 shows the relationship between the phase difference taken between water and ethanol, using Hilbert Transform and Fourier Transform (a) and (b), respectively, in (c) shows the distribution profile of each fringe pattern obtained, the rms error is 0.0763 rad.

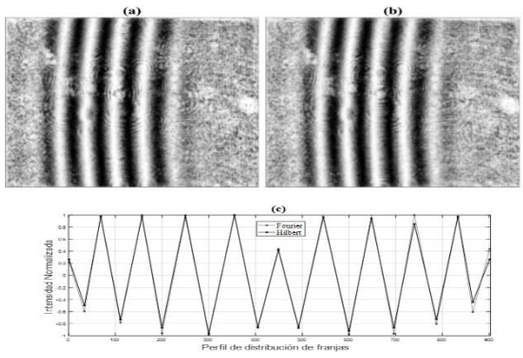

Figure 5 Phase difference water - ethanol, (a) Hilbert, (b) Fourier and (c) fringe pattern profile along the y-axis
Source: Own elaboration.

Applicability model

To evaluate the geometrical position change of the interference fringes, the differences between water and different values of immersion liquid (ethanol) were obtained, Table 1, the detection of the geometrical change was carried out with Hilbert Transform.

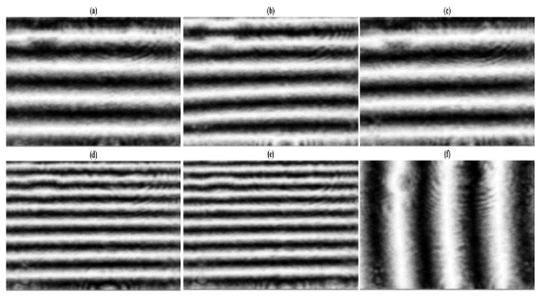

Figure 6. Geometric differences (a) ρ1- ρ2, (b) ρ1- ρ3, (c) ρ1- ρ4, (d) ρ1- ρ5, (e) ρ1- ρ6 and (f) ρ1- ρ7
Source: own elaboration.

Figure 6 shows the results obtained between each substance density change with a minimum value to detect phase change of 0.0099gr/cm^3 and maximum 0.1903 gr/cm^3 as indicated in Table 1, as the substance density changes, so does the amount of interference fringes, in Fig. 6 (a) and (f), the change of geometrical location of the fringes can be verified, for the case of (a), the optical phase change is at the x-position, for (f) the phase change is at the y-coordinate.

Conclusion

A procedure is described to determine the phase difference between two highly noisy, highly reflective, circular interferograms without the need for laborious techniques and using the Hilbert Transform. First, the spectral dependence between the Fourier Transform and the Hilbert Transform is shown, by comparing the phase difference with the conventional Fourier technique and the one proposed in this work, the results show an rms between the phase difference of two substances with different density of 0.0763 (water-ethanol), second, it is demonstrated that the interphase technique is capable of monitoring values in the order of microns without being affected by high frequencies as in the case of holography, obtaining a resolution in the measurement of 9.9-3gr/cm3. It can also be verified that by means of the phase difference of the analytical signals, it is possible to determine a change of substance, with the geometric rearrangement of the interference fringes.

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