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Presentation of the Content

In the first chapter we present, *Economic valuation of the impact on the Gulf of Mexico ecosystems: The deepwater horizon case* by PEREZ-VEYNA, Oscar, with adscription in the Universidad Autónoma de Zacatecas, as the next article we present, *Graphical interface for wind resource estimation in a region, case study: Cuatrocienegas, Coahuila de Zaragoza* by MERAZ-BECERRA, Fernando, SOLIS-CARDOZA, Víctor Manuel and CARRILLO-MARTÍNEZ, Jesús María, with adscription in the Universidad Tecnológica de La Laguna Durango, as the next article we present, *Calculation of liters to be sold at a gas station to optimize profits using the simplex method for a correct decision- making* by LERMA-LEDEZMA, David, CRUZ-SUSTAITA, Vianey and ESPINOSA-SOSA, Enrique Esteban, with adscription in the Universidad Politécnica de Altamira, as the next article we present, *Deduction and construction of a discrete probability function (Binomial and Geometric) with analytical development of expected value and variance* by GARCÍA-NAVA, Elizabeth, HERRERA-MIRANDA, Miguel Apolonio, HERRERA-MIRANDA, Israel and VILLAGÓMEZ-MÉNDEZ, Juan, with adscription in the Universidad Autónoma de Guerrero.

Content

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Economic valuation of the impact on the Gulf of Mexico ecosystems: The deepwater horizon case

Valoración económica del impacto sobre los ecosistemas del Golfo de México: El caso deepwater horizon

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Abstract

The value of nature does not escape the interests of those who cannot conceive the limits of such an important and vital element. The valuation on nature is developed to generate a reference and take actions accordingly. In this regard, the Contingent Valuation method is the most commonly referred to, but it involves the design of a structured survey in which the main question is whether there is a willingness to accept an amount of money in compensation (DAC) for the loss of an eco-friendly good or service. systemic or willingness to pay an amount of money to keep it (DAP). One or the other is defined without having a single referent and has been taken as a synonym of the total economic value. For this case, the problem of justifying the output value (DAP) was raised. The objective was to develop a methodology that based on data on household income and expenditure (ENIGH) in Mexico and applying a percentage to the expenditure per month, provided a basis for calculating that amount. The resulting methodology was accepted in the Valuation of goods and services provided by the Gulf of Mexico, after the Deepwater Horizon accident.

Contingent valuation, Deepwater Horizon, Gulf of Mexico

Resumen

El valor de la naturaleza no escapa a los intereses de quienes no logran concebir los límites de tan importante y vital elemento. La valuación sobre la naturaleza se desarrolla para generar un referente y tomar acciones en consecuencia. Al respecto, el método de Valoración Contingente es el más referido pero entraña el diseño de una encuesta estructurada en la que la pregunta toral es si hay disposición a aceptar una cantidad de dinero en compensación (DAC) por la pérdida de un bien o servicio eco sistémico o la disposición a pagar una cantidad de dinero para conservarlo (DAP). Una u otra se definen sin tener un referente único y se han tomado como un sinónimo del valor económico total. Para el caso, se planteó el problema de justificar el valor de salida (DAP). El objetivo consistió en desarrollar una metodología que a partir de datos sobre ingreso y gasto en hogares (ENIGH) en México y aplicando un porcentaje al gasto por mes, permitió disponer de una base para calcular esa cantidad. La metodología resultante fue aceptada en la Valoración de bienes y servicios que provee el Golfo de México, tras el accidente de la plataforma Deepwater Horizon.

Valoración contingente, Deepwater Horizon, Golfo de México

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Introduction

Establishing payment for ecosystem services (PES) is an essential step towards promoting environmental protection and building a society that is more aware of its relationship with nature. This has become relevant as we become aware of the serious consequences that human activity leaves in its wake on ecosystems that determine the stability of the climate and life on earth. Why have we lost sight of the importance of efforts to maintain the balance of nature? One reason for this is that the prevailing economic model on the planet seems insatiable in its identity, the process of accumulation and dispossession; its promoters did not find the intense use of cheap labour, the shrinking of the welfare state and the subjection of vulnerable sectors of the population to precarious conditions in housing, medical services, education, food, agricultural production and migration sufficient, and have now turned their attention to the goods and services provided by nature. This is the case of Latin American countries that are reaching the limit of their tolerance in their role as suppliers of raw materials with little or no added value, in order to seek, through capacity building and local investment, to make the most of their resources, but from the perspective of their own vision of development.

However, the critical situation has ceased to be located only in our continent and now we see how the insatiability of the economic model has moved to the geography of Central Asia, to show the world that this characteristic is not exclusive to neoliberalism, but also to the Soviet model, which goes beyond dispossession and shows the world the destructive and incomprehensible way of proceeding to take over territories, resources, people and nations.

There would be other interpretations of this last situation due to the interference of countries other than those directly involved in the conflict.

The commodification of ecosystem goods and services is largely explained by the transition from a classical economic model, which in its initial conception understood them as benefits to which a use value was associated, to a neoclassical economic model in which they are conceptualised as having an exchange value (Gómez-Baggethum *et al*., 2010).

Putting the market interest on the goods and services provided by nature has been a challenge that did not measure the consequences. Nevertheless, there are efforts that, from Environmental Economics, Ecological Economics, Economic Valuation and other disciplines, generate knowledge that contributes to rethink economic activity and try to steer our destiny towards achieving what seems to be our only way out, a sustainable society.

In particular, I would like to point out that after serious accidents related to oil spills, be it the sinking of large ships such as the Exxon Valdez off the coast of Alaska (1989), the Prestige off the Galician coast (2002) or the collapse of oil platforms such as Deepwater Horizon (2010) in the northern Gulf of Mexico and others, it is after these accidents that interest has arisen in making available the knowledge that is available to the public, is that the interest arises in having a technique that allows estimating the economic value of environmental goods and services as a way of having a reference for the evaluation of the economic impacts caused by accidents such as those referred to, to seek remediation and, where appropriate, the economic sanction that contributes to compensate the impact of externalities and thus contribute to induce a rational use of natural resources.

The global economy seems to be moving towards one that is characterised by a high dependence on energy, mainly oil and gas; recently, interest in lithium as a raw material to produce batteries for electric cars has been raised, and speculation about the location and size of the deposits is the subject of debates and state policies. However, the efforts for the development and use of clean energy do not seem to reach the power of hydroelectric, thermoelectric and nuclear energy.

In this context, it is important to understand the movement of crude oil around the world and the technology used for it; large oil tankers, pipelines, etc. The medium is a determining factor in the capacity and risk associated with the transfer function. The size of the crude oil spilled by a ruptured pipeline is not comparable to an accident involving an oil tanker on the high seas.

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In the latter case, approximately 81 accidents have been reported. (Libes, 2009). The highest frequency of accidents is located in the North Sea, the Mediterranean Sea, the Persian Gulf, the North Pacific, the Caribbean and the Gulf of Mexico. This dynamic generates a rational need that becomes a priority insofar as the global economy revolves around the availability of hydrocarbons, but the availability of hydrocarbons is not homogeneous; rather, it follows capricious routes that have allowed countries that have hydrocarbons in the subsoil or on the seabed to enjoy relatively comfortable economic situations, but not those that need to acquire them in a market that appears stable but also presents stages of high uncertainty, and with it, the value of crude oil.

Although technology has evolved to provide large oil pipelines, it is also recognised that there are physical limits that force the use of equipment, machinery, vessels, etc., which at some point give way due to the use and pressures to which they are subjected during the extraction, storage and transport operation, or due to human error; when this happens, we realise the magnitude of the implications thanks to the media, which report on the accidents and in some way help to raise awareness of the importance of events that were previously unknown. This work contributes in the sense of generating an option for the calculation of the exit value in a contingent valuation (CV) survey. The need consists of having valuation procedures that allow the circumstances associated with the type of claims mentioned above to be resolved.

To achieve this, it is possible to proceed by different techniques: Hedonic Pricing (PH), Travel Cost (CV), Contingent Valuation (VC). The VC technique is the most common; it was widely analysed and recommended by the Cordon Bleu panel convened by NOAA. (Carson T., 1997); However, the debate on the reliability of the VC technique is ongoing.

In the process of valuing goods and services provided by nature, it is important to be clear that the objective of the exercise is not to place them in a conventional market; the VC technique allows a market to be simulated and the exercise converges in the estimation of the total economic value.

Reference framework

Many exercises have been carried out by researchers and environmentalists to propose methods for estimating the economic value of the impacts of externalities on the goods and services provided by nature. An important experience was developed from the actions that Turkey decided to implement as a result of the repeated experience with British tourists arriving on its coasts. The research surveyed tourist users mainly on Olu Deniz beach, Turkey. The majority (70%) were British who used the beach for both recreational activities and to enjoy open spaces and scenery.

The main dislikes were found to be litter (41%), water quality (31%) and dog faeces (24%). Eighty-seven percent of British respondents expressed a positive willingness to pay (WTP). British tourists were willing to pay £1.03 per adult per visit. Significant variables in this determination were: age, gender, income level, social class and beach experience. The authors report a 10% decrease in tourists when an eco-tax was imposed in the Balearic Islands; this could mean a refusal to pay and is understood to be a market reaction to the increase or charge per beach visit (Blakemore & Williams, 2008).

This type of exercise is multiplied by several countries without any precision on the output value. Mexico is no exception. However, these experiences are minor in relation to the task that motivated the exercise of economic valuation of the impacts on ecosystems in the Gulf of Mexico. It was not only the magnitude, but also the consistency of the results because of the implications that could be derived from the econometric exercise.

Methodology

The output value to be proposed in the VC survey is decisive, as a Payment for Ecosystem Services (PES) project is not conceivable without the agreement between SE providers and buyers, which formalises compliance with the project as long as the established conditions are respected (Engel y otros, 2008).

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One of the important agreements in a PES programme is to assess the willingness of the parties to participate in estimating the willingness to pay (WTP) of the SE buyers who will support the suppliers and the willingness to accept compensation (WTP) for the SE sellers to implement the PES programme (conservation and sustainable practices programme) (Ureta *et al*., 2022).

However, it is important to note that, in this type of study, the dimension of the area of influence of the study should not be lost. It will not be the same to estimate the WTP or the WFD in a community of fishermen who carry out their activity in Boca del Río, Veracruz, as it will be in a study that includes the citizens of a nation. In the latter case, it will be complex to estimate one or the other parameter because of the great diversity that arises due to cultural differences, even though it is a country, between the inhabitants of the north and those of the south and centre, or between those who live on the coast and those who live in the mountainous or desert areas of the centre of a country.

Making a methodological proposal for such an important work in Religious Life cannot be absent from relating it to at least one specific case. To illustrate the proposal, the problem generated after the collapse of the Deepwater Horizon oil platform in April 2010 off the coast of Louisiana in the northern Gulf of Mexico will be taken to illustrate the proposal. The proposed method for determining the output value is based on four fundamental steps:

a) The most recent information on quarterly expenditure on food, beverages and tobacco in Mexican households is available.

b) Organise the information by deciles and disaggregate the monthly proportion.

c) On each decile, apply a percentage (0.5, 1, 1, 3, 3, 5 10%) which is supposed to be the proportion of expenditure that the respondent in the household would be willing to contribute. The resulting values are rounded to the nearest whole number (Table 1).

d) On the information per decile and per percentage applied, for the case of the double bounded approach, half and double are taken, according to the respondent's answer to the question Are you willing to contribute <quota> per month and for one year, to help in the protection of the ecosystem service?

	%				
Decil	0.5	1	3	5	10
I	$7.00	$13.00	$38.00	$62.00	$125.00
II	$8.00	$16.00	$47.00	$78.00	$155.00
III	$10.00	$19.00	$56.00	$93.00	$186.00
IV	$11.00	$21.00	$61.00	$102.00	$203.00
V	$11.00	$23.00	$67.00	$111.00	$222.00
VI	$13.00	$25.00	$75.00	$124.00	$248.00
VII	$15.00	$29.00	$86.00	$142.00	$284.00
VIII	$16.00	$32.00	$94.00	$157.00	$313.00
IX	$19.00	$37.00	$110.00	$184.00	$367.00
X	$26.00	$51.00	$153.00	$254.00	$508.00

Table 1 Percentage applied to monthly expenditure on Food and beverages in Mexican households 2010
Source: Own Elaboration

The reason to support the calculation of the output value (EV) <quota> in a percentage of the monthly expenditure in Mexican households on food, beverages and tobacco is no other than to the extent that the interviewer knocks at the door of the household, the head of household who accepts to collaborate will take as a reference when requesting his or her response on the amount selected from Table 2 based on the last and penultimate digit of the folio of the survey, will take as a reference, when asked to respond, the amount selected from Table 2 based on the last and penultimate digit of the survey folio, the amount of expenditure on food, beverages and tobacco, since it is considered that by deciding to contribute the proposed amount, this will be immediately translated as part of the expenditure indicated.

– This methodology provides a reliable basis on which it is possible to establish a key element in research on the estimation of the value of ES where the VC-Double bounded technique is applied.
– So far, the fee is established from the estimates of damage or impairment to the ES and the cost of remediation or reversal of the damage.

The table generated allows, by choosing according to the last and penultimate folio number of the survey, to set the figure to be proposed as the availability to pay (WTP) for the loss of enjoyment of ES. But this proposed value is calculated on the basis of the logic that is estimated to occur at the time of making decisions on spending in the household.

Last Digit of Folio Penultimate Folio Digit	Half	1,6 VS	double	Half	2,7 VS	double	Half	3,8 VS	double	Half	4,9 VS	double	Half	5,0 VS	double
1	3.5	7	14	6.5	**13**	26	19	**38**	76	31	**62**	124	62.5	**125**	250
2	4	**8**	16	8	**16**	32	23.5	**47**	94	39	**78**	156	77.5	**155**	310
3	5	**10**	20	9.5	**19**	38	28	**56**	112	46.5	**93**	186	93	**186**	372
4	5.5	**11**	22	10.5	**21**	42	30.5	**61**	122	51	**102**	204	101.5	**203**	406
5	5.5	**11**	22	11.5	**23**	46	33.5	**67**	134	55.5	**111**	222	111	**222**	444
6	6.5	**13**	26	12.5	**25**	50	37.5	**75**	150	62	**124**	248	124	**248**	496
7	7.5	**15**	30	14.5	**29**	58	43	**86**	172	71	**42**	284	142	**284**	568
8	8	**16**	32	16	**32**	64	47	**94**	188	78.5	**157**	314	156. 5	**313**	626
9	9.5	**19**	38	18.5	**37**	74	55	**110**	220	92	**184**	368	183. 5	**367**	734
0	13	**26**	52	25.5	**51**	102	76.5	**153**	306	127	**254**	528	254	**508**	1016

Table 2 Proposed output value in Mexican pesos <quota> for the double bounded model
Source: Own elaboration based on Gasto en Hogares Mexicanos en Alimentos, Bebidas y Tabacos (INEGI, 2010)

The table can be used as follows: If the enumerator randomly chooses a household from a given street in a city, he/she will notice that the questionnaire has a folio. Suppose it is 356. When the time comes, the interviewer asks: Would you be willing to contribute $11 per month for one year to a fund for the protection of the ecosystems of the Gulf of Mexico? Note that the value of $11 pesos is taken from table 2, noting that the penultimate digit of the folio is 5 and the last digit is 6, you will check the intersection of the fifth row with the first column.

Results

The Mexican state, through the then National Institute of Ecology (2012), issued the Terms of Reference to participate in the technical proposal for the Valuation of Environmental Services provided by the Gulf of Mexico (2012). This was in response to the state's own need for information from which it could take action against BP, in relation to the perception of Mexicans over the age of 18 regarding the effects on the Gulf of Mexico's ecosystems after the collapse of the Deepwater Horizon platform.

The INE technical committee and independent experts concluded that the most relevant proposal came from the Autonomous University of Zacatecas. After the definition, the activity to comply with the deliverables in due time and form began: 1). Biodiversity in the Gulf of Mexico 2). Methodology, 3). Results and discussion.

For the elaboration of the first two documents, things progressed without major setbacks; after the approval of the methodology and its implementation, the difficulties began. One of the components of the methodology, the structured questionnaire, took seven months to reach an agreement between the team responsible, INE and the team of independent experts. Once approved, doubts arose about the output value (EV) to be used for each interviewee. After discussions with the responsible persons in INE and the project director, it became clear that there was reasonable doubt about this VS. This paper demonstrates how important it is to have a sufficiently robust VS in the sense of having a consistent basis for calculation and in close relation to expenditure in Mexican households.

As explained in the methodology, a table was constructed that accompanied interviewers at all times and in all cities, and with which, based on the last two digits of the survey folio, the interviewer, when facing the person who agreed to collaborate by answering the survey, would choose the VS that would be proposed to the interviewee. This table is the contribution of this work and the basis of calculation is the data from INEGI-ENIGH (2010).

With the output value resolved, the stratified sample (n=3,200) was then drawn up, with 99% accuracy and reliability of .01. The logistics included the training of interviewers on the importance of the work and the relevance of clearly covering all the aspects indicated (use of visual materials, tone of the reading, not inducing the interviewee's response); the survey was carried out in the two most important cities in each state of the Mexican Republic. After a month of field work (Dec, 2012) in the country, a database was constructed that allowed, through the use of binary Logistic models, Mathematical Expectation, the procedure for the calculation of Confidence Intervals and the Double Bounded routine implemented in the STATA package, to estimate what Mexicans think is the value of the ecosystem services affected by the spill, under these four approaches.

Table 3 presents the methodology that guided the survey in 32 entities (64 cities).

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Study Universe	Mexicans aged 18 and over living in Mexico.
Sample size	3200 people aged 18 and over, distributed in a sample of 32 entities (64 cities).
Sample design parameters	Confidence level 99%, maximum error δ= .01
Unit of analysis	Household
Sampling method	Stratified at random with proportional decomposition
Questionnaire	Seven sections, 23 single- and multiple-choice questions
Data collection	Personal interview, graphic information session, questionnaire application, direct questions in the household.
Survey period	From 3 to 31 December 2012.
Field strategy	Training for survey service providers in the country.
Analysis	Statistics: descriptive, bivariate, multivariate, logistic regression.
Non-response treatment	Random behaviour is considered. Not replaced by any transformation.
Rejection rate	No problem, isolated case (Tijuana), not representative.
Software	SPSS, STATA, STATGRAPHICS

Table 3 Methodological window
Source: Own Elaboration

Once the database was designed and available, the parameters were estimated under the binary logistic regression model, by virtue of the nature of the dependent variable P14: Output value and the instruction expressed in the Terms of Reference. Since the possible answers were Yes or No, we have a categorical, binary variable.

Confidence interval estimation

The first approximation to the estimation of the value perceived by Mexicans aged 18 years and older in the stratified sample, of the value of the impact on the Gulf of Mexico ecosystems was developed from a model that considered the variables: p14 (Output value), Age, Income, p11QualityAmb, Schooling. The range of values produced by the model was projected over the universe of households in the Mexican Republic 28'159,374 (INEGI, 2023) (Table 4).

The second approximation was constructed from the calculation of the most likely value to occur (Mathematical Expectation) of the variable of interest, which is a point estimate based on the probability distribution of the possible outcomes of the P14 variables (Table 4).

The third approach to the value of the ecosystem impact was the one that expressly occupied the interest of the work; it consisted of the construction of several binary logistic models. Before presenting the results, it is pertinent to review the characteristics of the models used.

Models with qualitative dependent variable

A multiple regression model (not necessarily linear) allows us to explain the behaviour of a dependent variable Y as a function of a series of independent variables X_1, X_2, ..., Xk and an error term. ξi , i.e.:

$$Y = f(X_1, X_2, \ldots X_k, \xi_i), \; i=1,2,\ldots,k$$

In the particular case that the regression model is linear, we will have an expression of the form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \xi_i \qquad (1)$$

The objective of the regression will be to estimate the parameters of the model (in the linear case: β1, β2, ..., βk) so that the resulting model fits the data as well as possible. When the dependent variable is continuous, we proceed under a multiple linear regression model as above. In such a case the estimation of the parameters β1, β2, ..., βk is carried out using the Ordinary Least Squares (OLS) method.

It may happen that the dependent variable is a qualitative or categorical variable, i.e., it can only take a reduced set of values as is the case, when the interviewer asks the respondent if he/she agrees to support with <exit value>, the answer was only (Yes; No). In such a circumstance, the linear regression model is not suitable because of the nature of the response variable (it is not continuous). The choice will be logistic regression (Logit model).

It is important to remember that linear regression estimates the parameters by OLS (unbiased, minimum variance and consistent). (Infante & Zárate de Lara, 2013); in logistic regression, the parameters are estimated by the Maximum Likelihood (MV) method (biased, minimum variance) (Judge y otros, 1986).

The Logit model

One solution to the difficulties presented by the linear regression model in explaining the behaviour of a binary dependent variable is a Logit model of the form:

$$Y= f (\beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_k X_k + \xi_i)$$

The logistic function is expressed in the form:

$$f(z)= (exp(z))/ (1+exp(z)) \quad (2)$$

Therefore:

$$E(Y) = P(Y=1)=[exp (\beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots + \beta_k X_k)] / [1+ exp (\beta_0 + \beta_1 X_1 + \beta_2 X_2 +.. + \beta_k X_k)] \quad (3)$$

In this type of models it is not possible to directly interpret the estimates of the parameters β, as they are non-linear models. What is done in practice is to observe the sign of the estimators. If the estimator is positive, it will mean that increases in the associated variable cause increases in the probability of the desired event P14(Y = Si) occurring, (although we do not know the magnitude of these); if the estimator shows a negative sign, this will mean that increases in the associated variable will cause decreases in the probability of occurrence of the desired event, which in this case is P14(Y = Si). The latter expression represents the probability of a favourable response (Yes) to the WTP.

The following expression is the common form of a Logit model:

$$DAP= \frac{exp^{\beta}}{1+exp^{\beta}} \quad (3.1)$$

For the first model in table 4(c), the estimate by MV is:

β= 11.4844 - 0.00714539*p14 - 0.0207913*Age + 0.283374*Income=1 + 0.595038*Income=2 + 0.611854*Income=3 + 0.613429*Income=4 + 0.63947*Income=5 + 0. 691464*Income=6 + 0.65631*Income=7 + 0.458378*Income=8 + 1.10059*p11QualityAmb=1 + 0.728184*p11QualityAmb=2 + 0.830318*p11QualityAmb=3 + 0. 969274*p11AmbQuality=4 + 2.56101*p11AmbQuality=5 + 11.0014*Schooling=1 + 10.8948*Schooling=2 + 11.0819*Schooling=3 + 10.9141*Schooling=4 + 11. 418*Schooling=5 + 11.3148*Schooling=6 + 11.1519*Schooling=7 + 11.3955*Schooling=8 + 11.2963*Schooling=9 + 11.3351*Schooling=10 + 11.5046*Schooling=11

In the Logit model, other concepts are often used to further deepen the interpretation of the estimators. The Odds statistic is expressed by the following odds ratio:

$$Odds= [P(Y=Si)] / [1-P(Y=Si)] = [exp (\beta_0 + \beta_1 X_1+ \beta_2 X_2 +... + \beta_k X_k)] \quad (4)$$

Taking logarithms in the above expression, a linear expression for the model is obtained.:

$$Logit [P(Y=Si)]\equiv Ln \{ P(Y=Si) / 1- P(Y=Si)\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 +... + \beta_k X_k \quad (5)$$

In this case, the estimator of the parameter βi can be interpreted as the variation in the Logit term (the logarithm of the odds ratio) caused by a unit variation in the variable Xi (assuming the rest of the explanatory variables are constant).

Therefore, when reference is made to the unit increase in one of the explanatory variables of the model, the concept of Odds-ratio appears as the quotient between the two associated Odds (the one obtained after the increase and the one before the increase). If we assume that there has been a unit increase in the variable Xi, we will have that:

$$Odds\text{-}ratio = Odds1/Odds0= [exp (\beta i)] \quad (6)$$

From expression (6) it follows that a coefficient βi close to zero will mean that changes in the associated explanatory variable Xi will have no effect on the dependent variable and that an odds ratio close to one will indicate a significant effect on the probability of occurrence of P(Y=Yes) in the dependent variable due to the effect of the independent variable.

Under the above logic, the models of option c in Table 4 were constructed. No emphasis is placed on the significance of the specific contribution of each explanatory variable on the probability of a positive response P(Y=Yes).

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Therefore, the results focus attention on the estimation of that probability, on the understanding that the models (Table 4, item c) are the best performing ones.

The fourth option for the estimation of the impact value was developed from an econometric model with application of the doubleb subroutine in STATA software (Lopez, 2012) (Table 4).

Approach	Estimated annual WTP (pesos)	Remarks
a). Confidence interval approach to WTP estimation.	[9 695´467 772.00; 10 101´728 665.00]	
b). Approach to WTP estimation by Mathematical Expectation	12 688´275,108.00	Point estimate based on the probability distribution. In this case it is important to consider that 64% of the households declared their refusal twice.
c). Econometric models	Estimates of WTP/year	Variables involved
Model 1 Model 1 a (Yes Yes) Model 1 b (Yes No) Model 1 c (No Yes) Model 2 Model 2 a (Yes Yes) Model 2 b (Yes No) Model 2 c (No Yes) Model 3 Model 3 a (Yes Yes) Model 3 b (Yes No) Model 3 c (No Yes) Model 4 Model 4 a (Yes Yes) Model 4 b (Yes No) Model 4 c (No Yes)	Total DAP = 8 841´607,094.00 4 728´130, 669.00 3 456´133, 429.00 657´342 996.00 Total DAP = 9 112´003,386.00 4 869´372,825.00 3 627´442,442.00 615´188,118.00 Total DAP = 9 010´643,728.00 4 803´182,532.00 3 586´400,160.00 621´061,036.20 Total DAP = 4 026´306,293.00 2 053´559,760.00 1 537´128,060.00 435´618,473.20	p6, p7, p10, p14, age, income p3, p4, p5, p6, p7, p10, p11, p14, age, schooling, occupational status, income p5, p6, p7, p10, p14, age, income p14, age, income No information on environmental aspects is inserted in this model
d), Econometric model with application of doubleb subroutine in STATA	Total expected contribution per year 12,644´684,403.00	income, age, p6_4, p6_7, p6_9, p7_2, p7_6, p10_1

Table 4 Approaches to calculating the estimated economic value of the impact of the Deepwater Horizon collapse on Gulf of Mexico ecosystems
Source: Own elaboration based on survey data

Nomenclature

Q3: How has your contact with the beaches of the GoM been; Q4: How do you consider the environmental quality of the GoM to be; Q5: Which of the following people or organisations do you consider contribute most to the pollution of the GoM? Q6: Of the following sea-related problems, which of them are you most concerned about? Q6_4: Spillage from ship manoeuvring; Q6_7: Oil spill; Q6_9: Other. Q7:The GoM has marine ecosystems, which of these have you had contact with and what type of contact? Q7_2: Coral reefs; Q7_Sea grasses; Q10: From your point of view, of the following problems and their influence on the state of marine and coastal ecosystems, what priority attention should be given to each of them? Q10_1: Urban Growth; OccupySavermar: Your current occupation is related to the sea; Q14_Output value.

As can be seen (Table 4), the estimation of the economic value by the Expected Value technique and the Double Bounded technique was very similar. In third place, for the estimated amount, appears the technique of estimation by Confidence Interval, and the lowest estimates for the amount, were precisely those that yielded the binary logistic econometric models that are in the VC methodology, the reason for the project. The idea behind the breadth of approaches to value estimation has to do with the actions and negotiations that will undoubtedly arise when steps are taken in the legal, environmental and diplomatic arenas.

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Conclusions

The economic policy approach, the development of economic valuation schemes and the development of payment for ecosystem services schemes have contributed to placing the conservation of ecosystem services at the centre of the debate, but also to commodifying an increasing number of ecosystem services and reproducing the neoliberal economic paradigm and the logic of the market to address problems related to the environment.

Exercises of the magnitude described here will enable the Mexican scientific community and society in general, the native communities, but above all the Mexican state, to have the necessary knowledge, hitherto absent, of a method for estimating the value of the services provided by nature, so that the latter can act accordingly. It will be more important to have developed a unique experience in terms of its magnitude, from which much was learned, especially given the seriousness of the losses caused by human activity and the lack of knowledge about the methods for valuing environmental goods and services. It is also important to note that the delay in publishing the results is due to the Terms of Reference and to the fact that the author is now authorised to openly disclose the estimates made.

After this type of study, consequent actions are beyond the scope and responsibility of the author. These are the elements available to the federal authority to act as they deem appropriate.

Finally, it is important to reiterate that the assignment only included the approach of econometric models, however, from the experience gained from the valuation practice, it is well known that it will always be better to have more than one approach to estimate the economic value of the subject in question, experience that is expressed in the complementary estimates. This breadth of procedures gives room for negotiation processes, which go beyond the domain of work and enter the field of diplomatic relations, in which the rigorous vision of the environmental movement is always in force, in order to avoid inaction after such a visible loss and from which unprecedented experiences were derived, which cannot remain in the writing of a work of this nature.

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Graphical interface for wind resource estimation in a region, case study: Cuatrocienegas, Coahuila de Zaragoza

Interfaz gráfica para la estimación del recurso eólico en una región, caso de estudio: Cuatrocienegas, Coahuila de Zaragoza

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Abstract

Mexico has an estimated wind potential of 70 GW, however, until 2022 only 7.3 GW were used. This is due, among other factors, to the lack of analysis of the wind resource in many of the country's regions, especially in those where the wind potential is not obvious. With the aim of promoting wind power throughout the country, a free application has been developed that analyzes the resource in detail in any region using the annual records of its wind speed and direction. To demonstrate its functionality, data from the year 2022 from Cuatrocienegas, Coahuila de Zaragoza were used, obtaining the average energy density per hour (MED) using two analytical methods and the Weibull probability distribution function (WPDF). The instantaneous power curve, the histogram of the occurrence of the wind and the wind rose that shows the predominant direction of the wind and its trend of change were also presented. In conclusion, the developed application provides the necessary information to evaluate the technical feasibility of installing horizontal axis wind turbines in the analyzed region.

Wind resource, WPFD, MED

Resumen

México tiene un potencial eólico estimado de 70 GW, sin embargo, hasta el año 2022 solo se aprovechaban 7.3 GW. Esto se debe entre otros factores, a la falta de análisis del recurso eólico en muchas de las regiones del país, especialmente en aquellas donde el potencial eólico no es evidente. Con el objetivo de impulsar el aprovechamiento eólico en todo el país, se ha desarrollado una aplicación gratuita que analiza detalladamente el recurso en cualquier región utilizando los registros anuales de su velocidad y dirección del viento. Para demostrar su funcionalidad, se utilizaron datos del año 2022 de Cuatrocienegas, Coahuila de Zaragoza, obteniendo la densidad de energía media por hora (DEM) mediante dos métodos analíticos y la función de distribución de probabilidad de Weibull (FDPW). También se presentaron la curva de potencia instantánea, el histograma de la ocurrencia del viento y la rosa de los vientos que muestra la dirección predominante del viento y su tendencia de cambio. En conclusión, la aplicación desarrollada proporciona la información necesaria para evaluar la factibilidad técnica de instalar aerogeneradores de eje horizontal en la región analizada.

Recurso eólico, FDPW, DEM

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I. Introduction

Electricity generation is the second largest human activity producing CO2, SOx, NOx, CO, PM10 and PM2.5 in Mexico (SyD, 2019). Specifically, in 2019, electricity generation accounted for 17% of total CO2 emissions in Mexico (Evalua, 2022). For this reason, this industry is one of the human activities that alter the greenhouse effect causing global warming (International Energy Agency, 2016a; IEA, 2016b; IEA, 2016c; REN21, 2016; IEAd, 2016). Although current electricity systems globally have been able to meet energy demand for the past 70 years, demand is soon expected to outstrip energy production (Farhangi, 2010).

In 2022, 73.9% of electricity production in Mexico will be from fossil fuel-based power plants. Combined cycle, conventional thermal, turbogas and coal-fired power plants accounted for 58.1%, 6.3%, 4.6% and 4.3% of this percentage, respectively. While the use of renewable energies such as hydro, wind, solar, nuclear, geothermal and biomass was limited to 10.7%, 6.1%, 4.9%, 3.2%, 1.3% and 0.03%, respectively (CIPP, 2023).

Since 2014, the distribution between the use of renewables and oil derivatives had not changed significantly. To address this problem, Mexico undertook a series of energy, political, economic and social reforms with the aim of enabling the significant introduction of renewable energy power generation systems (Cancino et al., 2011).

These reforms were cancelled, however, Mexico will comply with the commitments agreed in the 2015 Paris Convention, generating 35% of electricity from renewable energy by 2030 (Lopez, 2022).

In Mexico, electricity generation from renewable energy is mostly supported by hydropower and wind energy (CIPP, 2023; Murthy & Rahi, 2017).

Wind energy in Mexico is abundant, has a high energy density, its exploitation is viable and it is widely distributed in the territory, which makes it one of the best alternatives for electricity production (Pérez-Denicia *et al*., 2017).

However, until 2021, only 7312 MW of the total wind potential, which has been estimated at around 70 GW (MEP, 2023), was being exploited in Mexico. By May 2022, only 15 of the 32 Mexican states will have partially harnessed their wind resources. The installed capacity in Oaxaca (2758 MW), Tamaulipas (1725 MW) and Nuevo León (793 MW) stands out, accounting for 72% of the total in the country (AMDEE, 2023). While states such as Yucatán, Veracruz and Coahuila de Zaragoza, which also have a substantial wind resource, do not have the same level of development (NREL, 2022).

This problem is due, among other factors, to the lack of wind resource characterisation in many of the country's regions. Efforts for the detailed analysis of the wind resource in the country have focused on areas with obvious wind potential that allow the installation of high-power horizontal axis wind turbines. This has led to a lag in the exploitation of the wind resource in regions with little obvious potential (NREL, 2022). In order to contribute to the exploitation of the wind resource in the whole country, and especially in those regions with a non-obvious wind potential, this article shows a final graphical interface developed for a web application that allows a detailed analysis of the wind resource in any region.

This application will require wind speed and direction data for the area of interest recorded at a height of 10 metres, every 10 minutes, for at least one year. For the development of this application, Matlab software was used for data processing (back-end), while the C# .NET programming language was used for data capture and as a graphical interface (front-end), in conjunction with the tools provided by the Microsoft Visual Studio suite. The rest of the paper is organised as follows: Section II describes the three methodologies used to calculate the DEM, as well as the process of creating the polar graph that describes the predominant wind direction in the region of interest; Section III deals with the development of the web application used to calculate and display the elements described in the previous section; Section IV presents the results obtained from the analysis of the wind resource of the region taken as a case study to evaluate the functioning of the graphical interface . The work concludes with Section V.

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II. Methodologies used for wind resource assessment in any region

The wind energy potential can be estimated from an exact empirical method or by numerical methods that approximate the parameters of interest in the region. The former involves the analysis of the terrain topography and its roughness coefficient, the local temperature and pressure and the wind speed to obtain the mechanical wind power which in conjunction with its direction will determine the wind energy potential of the wind. The second involves the use of probability density functions (PDF) such as the Weibull to estimate the wind behaviour and thus the wind potential of the region.

2.1. Mean Energy Density Extracted from the Wind Obtained through the Root of the Mean Cubic Velocity (DEM_VRMC).

2.1.1. Instantaneous power of the free air stream.

The kinetic energy in air of mass m moving with velocity V, is (Patel, 2006):

$$Ec = \frac{1}{2}\, m\, V^2 \tag{1}$$

The power available in a free air stream is the flow of kinetic energy per unit time through the cross-sectional area of the wind turbine rotor blade (Patel, 2006):

$$Pa = \frac{EC}{t} = \frac{1}{2}\frac{m}{t}\, v^2 = \frac{1}{2} M v^2 = \frac{1}{2}\rho\, A\, v^3 \tag{2}$$

where P is the instantaneous mechanical power of the moving wind (w), M is the mass flow rate (Kg/s), ρ is the air density (Kg/m^3), A=πr^2 is the area swept by the rotor blades (m^2) and v is the air speed. (m/s).

2.1.2. Instantaneous Power in the Wind Harnessed by a Wind Turbine

The instantaneous power extracted by the rotor blades is obtained from the difference between the incoming wind v and the outgoing wind. vo (Wais, 2017):

$$P = \frac{1}{2} M(v^2 - vo^2) \tag{3}$$

Macroscopically, the wind speed is discontinuous from v to vo in the plane of the rotor blades, averaging approximately (v+vo)/2, thus:

$$M = \rho\, A \left(\frac{v+vo}{2}\right) \tag{4}$$

The instantaneous mechanical power extracted by the rotor, which drives the electrical generator, is therefore:

$$P = \frac{1}{2}\rho A \frac{\left[\frac{1}{2}(v+vo)\right]}{(v^2 - vo^2)^{-1}} = \frac{1}{2}\rho A v^3 \left[\frac{\left(1+\frac{vo}{v}\right)\left[1-\left(\frac{vo}{v}\right)^2\right]}{2}\right] \tag{5}$$

$$P = \frac{1}{2}\rho\, A\, V^3 Cp \tag{6}$$

where Cp is the power coefficient and represents the fraction of the incoming wind power that is extracted by the rotor blades (Wais, 2017). In this article a value for Cp of 0.4 was assumed, which is typical for commercial wind turbines (THE WIND POWER, 2023).

2.1.3. Root Mean Cubic Velocity

The monthly wind speed varies around ±30% to±35% above the average wind speed at a typical location during the year (Patel, 2006). Therefore, the wind speed used to determine the power density in (6) should be (Pishgar-Komleh *et al*., 2014):

$$V_{rmc} = \left(\frac{1}{n}\sum_{i=1}^{n} vi^3\right)^{\frac{1}{3}} \tag{7}$$

Finally, the average energy density extracted from the wind (DEMV) will be obtained in a period that will depend on the quantity and frequency with which the measurements have been made (Patel, 2006):

$$\frac{E}{A} = \frac{1}{2}\rho\, {V_{rmc}}^3 Cp \tag{8}$$

2.2. Mean Extracted Wind Power Density from the Weibull Density Probability Density Function (DEM_FDPW)

2.2.1. Weibull PDF

The PDF indicates the probable frequency at which the specified velocity will occur in the study region. The Weibull PDF is given by (Murthy, 2017; Patel, 2006; Ozat & Celiktas, 2016; Wo *et al.*, 2011):

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$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}} \quad (9)$$

where v is the wind speed (m/s), k>0 is the shape factor (dimensionless) and c>0 is the scale factor (m/s).

2.2.2. Weibull Cumulative Distribution Function

The cumulative distribution function is the accumulation of relative frequency of each wind speed interval, it is defined by (Murthy, 2017; Patel, 2006; Ozat & Celiktas, 2016; Wo *et al*., 2011):

$$F(v) = \int_0^v f(v)dv = 1 - e^{-\left(\frac{v}{c}\right)^{k}} \quad (10)$$

2.2.3. Estimation of the Parameters of the Weibull PDF

There are at least 15 methods to estimate the c and k parameters of the Weibull PDF. In this paper only the four most common methods will be presented: Justus standard deviation, MDEJ (Justus *et al*., 1977), Lysen standard deviation, MDEL (Lysen, 1983), simplified moments, MMS (Azad *et al*., 2014), and probability weighted moments, MMPP (Usta, 2016). Table 1 summarises the equations used in each method to estimate by the parameters c and k.

	Shape Parameter (k)	Scale Parameter (c)
MDEL	$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086}$ (11)	$c = \frac{\bar{v}}{\left(0.568 + \frac{0.433}{K}\right)}$ (15)
MDEJ		
MMS	$k = \left(\frac{0.9874\,\bar{v}}{\sigma}\right)^{1.0983}$ (12)	$c = \frac{\bar{v}}{\Gamma\left(1+\frac{1}{k}\right)}$ (16)
MMPP	$k = \frac{\ln(2)}{\ln(\bar{C})}$ (13) $\bar{C} = \frac{\bar{v}}{\frac{2}{n(n-1)}\sum_{i=1}^{n} v_i(n-1)}$ (14)	

Table 1 Equations for determining the parameters of the Weibull WDF for the four methods used

2.2.4. Characteristic Wind Speed Values Using the Weibull WTPF

Knowing the Weibull parameters, the root mean cubic velocity, the mean cubic velocity, the most probable wind speed and the highest wind speed can be calculated from equations 17, 18, 19 and 20, respectively. (Justus, 1977; Akdag & Guler, 2015; Christofferson & Gilette, 1987).

$$V_{rmc} = \int_0^{Vmax} v * f(v)\, dv \quad (17)$$

$$V_{rmc}^3 = \int_0^{Vmax} v^3 * f(v)\, dv \quad (18)$$

$$V_{mp} = c\left(1 - \frac{1}{k}\right)^{\frac{1}{k}} \quad (19)$$

$$V_{max} = c\left(1 - \frac{2}{k}\right)^{\frac{1}{k}} \quad (20)$$

2.2.5. Energy Density with the Weibull PDF

Substituting equation 18 into equation 8, the following equation is obtained:

2.3. Average Extracted Power Density from Wind Obtained by Instantaneous Power

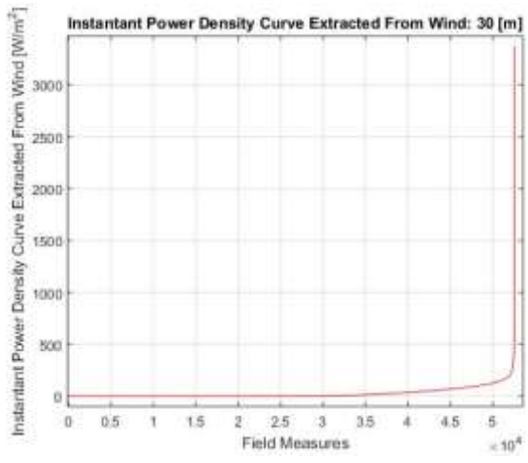
Curve Integration (DEM_IPI).

1. The instantaneous power density in the wind (IPD) is calculated from each of the 52560 wind speeds recorded by the automatic meteorological station (AMS) every 10 minutos durante al at least one year in the region of interest, as expressed in Equation 22.

$$DPI_i = \frac{1}{2}\rho\, {v_i}^3 Cp \quad (22)$$

where v_i is the ith wind speed recorded in the region.

2. The annual instantaneous power density curve for the region of interest is generated. An example of this power curve is shown in Figure 1.



Graph 1 Annual wind power density curve

3. The curve shown in graph 1 is numerically integrated with respect to time to obtain the total wind energy density for the year (DEVT):

$$DEVT = \left(\frac{1}{2}DPI_1 + \sum_{i=2}^{n-1} DPI_i + \frac{1}{2}DPI_n\right)T \quad (23)$$

4. Finally, by dividing the DEVT by the corresponding factor, the DEMV in a given period is obtained. Equation 24 determines the hourly DEVT from a sample of data recorded every minute for one year.

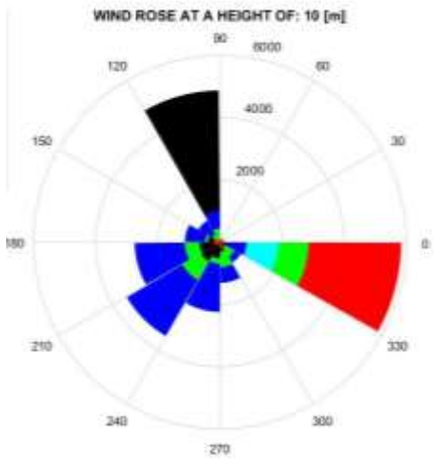
$$\frac{E}{A} = DEVT\left[\frac{W*año}{m^2}\right]\left[\frac{365\ dias}{1\ año}\right]\left[\frac{24\ h}{1\ día}\right] = \frac{DEVT}{8760}\left[\frac{w*h}{m^2}\right] \quad (24)$$

2.4. Wind Rose

It is a polar chart typically divided into 12 sectors representing ranges of wind directions.

$$\frac{E}{A} = \frac{1}{2}\rho\ Cp \int_0^{Vmax} v^3 * f(v)\ dv \quad (21)$$

Within these wind direction ranges, the annual frequency of 12 wind speed ranges is shown using colours. Sometimes, instead of frequency of occurrence, percentage of occurrence of each wind speed range is shown. An example of a wind rose is shown in figure 2.



Graph 2 Wind rose

Even if the wind in a region has a high average energy density, if the wind direction is not constant, it will not be suitable for the installation of wind turbines. This is because regions with a high turbulence rate (fluctuation in wind direction) cause wind turbines to be constantly rotating so that the swept area formed by the blades is oriented perpendicular to the wind.

III. Web Application

Web applications play a key role in the development of modern scientific applications. In this article, we explore the advantages of using a graphical interface created in C#.NET, an object-oriented programming language and development platform widely used in industry. C#.NET offers a robust development environment, design facilities and broad compatibility that make it an ideal choice for creating GUIs in scientific applications C#.NET provides in terms of performance, interoperability and access to scientific libraries, in the particular case of this article, for data analysis, C#. Net offers a library capable of interacting with the Matlab numerical calculation software. The final interface of the web application can be seen in Figure 3.

Graph 3 Developed graphical interface

The web application uses multiple methods developed in Matlab to calculate the wind DEM because it adapts to the available data that the user has for the region. The analytical methods based on the integration of the instantaneous power curve (DEM_IPI) and the mean cubic velocity (DEM_VRMC) are accurate, but require the recording of the velocities in the region every 10 minutes for a year, information that is not always available. While the methods based on the Weibull PDF are approximate numerical methods that in some cases require only a few data such as the average annual wind speed.

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IV. Results

4.1. Region Analysed

The functionality of the graphical interface was tested using wind speed and direction data recorded every 10 minutes by the EMA Cuatrocienegas located in the state of Coahuila de Zaragoza (Latitude: 27.002, Longitude: -102.073) during the year 2022.

4.2. Average Wind Energy Density in Cuatrocienegas, Coahuila de Zaragoza

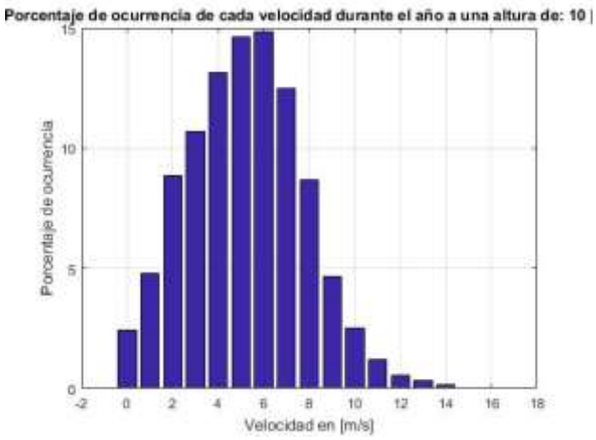
For the study region, the DEM obtained by the DEM_IPI and DEM_VRMC methods was 486.7 and 493.9 kWh, respectively. Where the DEM_IPI is considered the real value of the DEM and will be the reference to evaluate the accuracy of the other methods used in the GI. The MDEJ, MMS and MMPP methods presented an acceptable level of pressure with errors of 2.73%, 1.11% and 1.48%, respectively. On the other hand, MDEL presented an error of 57.4%. Table 2 summarises the above.

Method		Result [kWh/m^2]	Error [%]
DEM_IPI		486.7061	-
DEM_VRMC		493.8962	1.48
DEM_FDPW	MDEL	766.0859	57.4
	MDEJ	499.9966	2.73
	MMS	492.0928	1.11
	MMPP	493.8962	1.48

Table 2 Value of DEM obtained by various methods

4.3. Wind Behaviour

To determine the wind resource of a region it is not enough to determine the DEM, it is also necessary to analyse wind trends and wind direction. Graph 4 shows the percentage of occurrence of each wind speed, with speeds of 5 m/s and 6 m/s being the most frequent with 14.9% and 15% of the total recorded. From this graph it can also be deduced that the distribution of the frequency of occurrence of the speeds resembles a Gaussian bell, with most of the data concentrated in the centre of the curve and close to the value of the average speed.

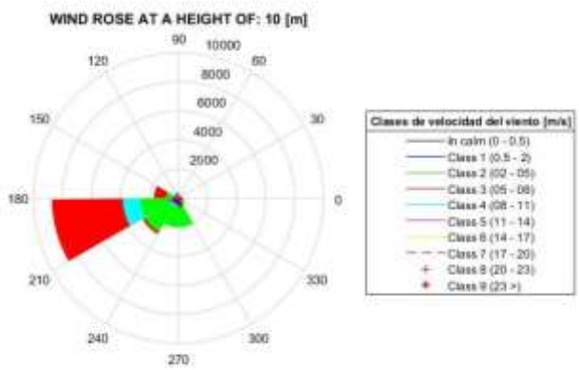


Graph 4 Percentage of annual occurrence of each wind speed

As mentioned in section 2.4, the wind rose allows to determine how stable the wind direction is throughout the year. Because even if the wind speed is high, if the wind speed is turbulent it will not be usable.

Low-power wind turbines tend to have a fixed orientation with their swept area perpendicular to the wind direction with the highest percentage of occurrence during the year. While medium and high power wind turbines, although they have a rotation system to follow the wind direction if the wind is very changeable, the wind turbine will consume much of the energy produced in this rotation making its operation deficient.

Graph 5 shows that wind speeds of 5 to 8 m/s (which have the highest frequency of occurrence) flow from the west throughout the year, as do wind speeds of 8 to 11 m/s.



Graph 5 Wind rose of the studied region

Acknowledgements

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Conclusions

According to the results obtained, it is possible to affirm that the application developed to determine the wind resource of a region of interest works correctly, offering the necessary information to determine the technical feasibility of the installation of a commercial wind turbine of any power.

The areas of opportunity to be covered are expected to add more methods for the estimation of the DEM from the Weibull FDP, the ability to extrapolate the results to different heights even without real measurements at those heights, the calculation of the instantaneous power from a specific and not approximate value for the air density, and the determination of the Weibull parameters with an acceptable accuracy when the frequency distribution of the wind speed is multimodal.

Finally, it is concluded that it is technically feasible to install a commercial wind turbine of low or medium power in the Cuatrociénegas region, whose typical start-up speed is 3 m/s.

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Calculation of liters to be sold at a gas station to optimize profits using the simplex method for a correct decision- making

Cálculo de litros por vender en una gasolinera para optimizar las ganancias utilizando método simplex para una correcta toma de decisions

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Abstract

Measurements of monthly sales were made from January to June 2021 at a gas station in order to obtain historical information and based on it determine, applying the simplex method for decision making, the correct number of liters of gasoline of the Magna (green) and Premium (red) type and Diesel necessary to obtain a maximum profit based on a budget that the company has. The monthly observations and the calculation were made using the EXCEL software with pre-recorded formulas to apply the simplex method and were developed during the period from July 05 to 09, 2021. This research work takes advantage of the benefits of the simplex method to find and optimize the best cost-benefit relationship and find the maximum possible profit in the sale of gasoline at a given station. The fluctuation in the changes of the gasolines costs in the last three years was considered to carry out the calculations of this investigation.

Measurements, Optimize, Fluctuation

Resumen

Se realizaron mediciones de las ventas mensuales desde el mes de Enero al mes de Junio de 2021 en una estación de gasolina con el objeto de obtener información histórica y en base a ella determinar, aplicando el método símplex para toma decisiones, el correcto número de litros de gasolina del tipo Magna (verde), Premium (roja) y Diésel necesarios para obtener una ganancia máxima en base a un presupuesto con el que cuenta la empresa. Las observaciones mensuales y el cálculo se hizo utilizando el software EXCEL con fórmulas pregrabadas para aplicar el método símplex y fueron desarrolladas durante el periodo del 05 al 09 de Julio de 2021. Este trabajo de investigación aprovecha las bondades del método símplex para encontrar y optimizar la mejor relación de costo beneficio y encontrar la máxima ganancia posible en la venta de gasolinas en una determinada estación. Se consideró la fluctuación en los cambios de las gasolinas en los últimos tres años para realizar los cálculos de esta investigación.

Mediciones, Optimizar, Fluctuación

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Introduction

This research paper shows the results of the application of a decision-making method (simplex method) based on historical sales data of three products (magna gasoline, premium gasoline and diesel) offered by a gas station.

The main idea of it is to find a point of balance between the number of liters sold per month of each of the products and to obtain, based on this, an optimzation in sales, understood as optimization, the adequate number of each one of them to obtain the highest monthly profit and keep track of the liters sold of each of them.

1. Decision making

Decision making is a daily process in which many times the decisions that are made consciously or unconsciously. It is neccesary to define the concept as well as the identifiction of some elements that influence the decision making, such as experience, knowledge and intuition.

In the words of (Pérez Callejo, 2015) decision- making is a structured process of decomposition of the possible solution alternatives and is carried out based on one or several axiological criteria where each of that aletrnatives is assigned a specific weight.

Two of the elements to differentiate when a making decision is made are the states of nature and the alternatives or options. In the states of nature, the decision maker (who makes the decision doesn´t have any injerence in the decision due it is a situation out of his control while in the alternatives or options he has full control to select any of them).

Intuition in decision making can change the decision making toward the subjectivity since it's based on what the decision maker feels, imagines or believes according to his criteria.

This subjectivist behavior must be replaced by objectivity based on concrete and hard facts provided by a mathematical model.

In the opinion of (Jiménez Lozano & Jiménez Muñoz, 2012) some models for decision making are decisions in certainty environments, decisions in risk environments, in uncertainty environments with experimental information. One of the main objectives that are sought with the correct decision making through any of the methods described above is for an optimization, which will be discussed in the next section.

2. Optimization

Many real-life problems are solved when the researcher manages to translate the observed problem into a mathematical model, that is, to identify variables, values and restrictons, as well as the relations of operations between them that are conducive to establishing an equation, and with it, finding an unique value that satisfies all the conditions established in the problem analysis stage.

However, it isn´t always a value that can satisfy all the conditons or restrictions, but it has been observed that a set of values can also satisfies them. Contrary to what could be thought of as a success, this sometimes represents a second problem because now it's neccesary to identify which of this set of values is the one that gives us the best solution and this value is known as the optimal value.

It's important to specify that optimization is generally understood as the fact of finding a greater profit and often the other sense of optimization, which consists of reducing costs (minimization), is left aside. That is why, when defining the problem, it must also be established whether the problem to be solved is one of maximization or minimization

For the present research work, we´ll work according to the logic of maximization, since what is intended is to find the greatest possible profit from the sales of the products at a gas station. In optimization problems, it's desired to achieve a set of objectives with a maximum benefit or a minimum cost, adjusting the problem to the reality through a set of restrictions as well as the inclusion of a goal called the objective function (Santisteban Urquiza, 2012). Many of these types of existing problems are solved by applying inexact methods and it's precisely in the type of exact methods where the simplex method is located, which we'll address in the next section.

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3. Simplex method

The simplex method is a method that works based on cycles or repetitions of operations. Each time a loop is performed, a better solution is found.

In real life, the first cycle will not give an optimal solution since it's assumed that it can always be improved. In many cases, this first solution is 0 with negative values in the first row as a sign that the system should be improved through a second iteration.

Each cycle or iteration will find a better solution, that is, the third solution impoves the second one and the fourth one improves the third one and so on.

The loop ends when there are no more negative values (all positive or 0) in the first row of the simplex table and at that moment the best possible solution has been found.

The simplex method has the added value with respect to its predecessor, the graphical method, in that it works for n variables and m number of restrictions, which makes it more versatile.

The problem to solve consists of finding the optimal combination of liters sold of the three products that are sold at the gas station to get the maximum profit and thus keep track of the liters that are sold in the sense of prioritizing in the sale of liters according to the results of the application of the simplex method.

The simplex method is also known as the self- directed optimization method (Ahumada & Guerrero, 2010), which is based on measuring the effects that produce changes in the variables and deducing from the results what must be done in the succesive changes to get improvements in the behavior of the phenomenon being studied.

Regarding the simple sequential optimization method, the following steps are established (García Soto, Rodríguez Niño, & Trujillo, 2013)

- The vertex that has the worst response is reflected over the centroid of the remaining k vertices ($\alpha = 1$).
- If the response of the new vertex is better than that obtained in previous vertices, an expansion of this new vertex is applied ($\gamma = 2$, expansion coefficient) to extend the movement in the direction of reflection.
- If the new vertex response is better than the previous worst vertex, but not better than the resulting vertices, $0 < \beta < 1$ is applied to reduce the step size for the new cycle.

4. Operation of gas stations

Regarding the operations of gasoline outlets, they are conceived as a point of sale of gasoline for the supply of motor vehicles and is made up of several gasoline vending machines, storage tanks for diferente types of gasoline, and space for the service an supply of gasoline to vehicles. (Ramírez, Patete, & Javier, 2011).

However, regarding the fluctuations of gasoline prices in Mexico (Ibarra Salazar & Sotres Cervantes, 2008), it's known that these are regulated by the federal government through the Ministry of Finance and Public Credit (SHCP) and are mainly based on the fiscal aspect, the uniform price policy per volumen unit throughout the country and differential prices policies in the northern and southern regions of the country It's a fact that gasoline moves a country regardless of the new energies that are emerging or that are about to be released the reality is today, gasoline is the main input that moves the automobile and transport industry (land and air), both, in Mexico and in the world.

According to (Canobbio Rojas & Cárdenas Aragón, 2020), for the operation of gas stations in Mexico, it's necessary to point out that gas stations must belong to a mandatory scheme called SASISOPA (Industrial Safety and Environmental Protection Administration System), which was created to regulate the activities of the hydrocarbon sector (MX ASEA 2019).

5. Metodology to develop

The historical files in Excel that the company has were searched and based on this, the simplex method was applied with the following objective function of maximization and whose restrictions according to the needs of the gas station as follows:

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– The variable *x1* will represent the number of liters of magna gasoline (green).
– The variable *x2* will represent the number of liters of premium gasoline (red).
– The variable *x3* will represent the number of liters of diesel (black).

The current prices of the products with their respective earnings are as follows (Juy, 2021). See Table 1.

Product	Cost per liter	Sale per liter (public)	Revenue
Magna gasoline	$18.61	$20.23	$1.62
Premium gasoline	$20.41	$22.18	$1.77
Diesel	$19.62	$21.39	$1.77

Table 1 Current prices
Own Elaboration

In discussions with the managers of the gas station, it was established that the sum of the 3 products should not exceed 1,000,000 liters per month and that the profit from premium gasoline is expected to be at least double the sum of the liters of the other 2 products (magna and diesel).

Also, derived from saids talks, a monthly budget of $19,453,000 was established, which should not be exceded at any time. With this information and with the individuals earnings of each one of the products specified in Table 1, the following mathematical model was estabilised:

Objective function (profit maximization)

$Max\ z$=1.62 $x1$+1.77 $x2$+1.77 $x3$

Restricted to:

$x1 + x2 + x3 \leq 1{,}000{,}000$
(not exceed 1,000,000 monthly liters)

$18.61\ x1 + 20.41\ x2 + 19.62\ x3 \leq \$19{,}453{,}000$
(never exceed the Budget of $19,453,000)

$x2 \geq 2\ (x1 + x2)$
(premium profit at least double of the other two products)

$x1, x2, x3 \geq 0$
(conditions of non-negativity)

The mathematical model is summarized in the simplex table shown in Table 2, thus establishing the first solution.

Var	z	x1	x2	x3	h1	h2	h3	Revenue
Z	1	-1.62	-1.77	-1.77	0	0	0	0
h1	0	1	1	1	1	0	0	1,000,000
h2	0	18.61	20.41	19.62	0	1	0	19,453,000
h3	0	-2	1	-2	0	0	1	0

Table 2 Tabla Initial simplex table with the first solution
Own Elabration

This is a first solution, but obviously, it isn't the optimal one since there is no profit.

The most negative variable (greatest negative value) is chosen as the variable entering the system. In this case, it makes no difference to choose *x2* or *x3* since both have the same value,-1.77. The variable *x2* was chosen.

Once the variable that enters the system is determined, the variable that will leave the system is determined as follows.

The following operations are carried out to determine the variable that will leave the system, which will be the one with the lowest value in the OPERATIONS column, being the slack variable *h3*, see Table 3.

1,000,000 / 1 = 1,000,000
19,453,000 / 20.41 = 953,111.22
0 / 1 = 0

Var	z	x1	x2	x3	h1	h2	h3	Revenue	Operactions
z	1	-1.62	-1.77	-1.77	0	0	0	0	
h1	0	1	1	1	1	0	0	1,000,000	1,000,000
h2	0	18.61	20.41	19.62	0	1	0	19,453,000	953,111.22
h3	0	-2	1	-2	0	0	1	0	0

Table 3 Middle table
Own Elaboration

At the intersection of the variables *x2* and *h3* is the pivot with a value of 1. This pivot serves as the basis for searching for the appropriate operations and leaving the entire column of *x2* at zeros (0) with the operations that are included in the column of operations and Table 4 is obtained with the second solution.

Var	z	x1	x2	x3	h1	h2	h3	Revenue	Operations
z	1	-5.16	0	-5.31	0	0	1.77	0	
h1	0	3	0	3	1	0	-1	1,000,000	R1 + 1.77*R4
h2	0	59.43	0	60.44	0	1	-20.41	19,453,000	R2 – R4
x2	0	-2	1	-2	0	0	1	0	R3 – 20.4*R4

Table 4 Second solution
Own Elaboration

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This is a second solution, but again an earn of 0 is detected, which means that is not yet the solution. Since there are 2 negative values in the *x1* and *x3* columns, it's known that one more iteration must be performed to find a new solution.

For this new solution, the variable that will enter the system is chosen, being the most negative (*x3*). Now, to determine the variable that leaves the system the operations specified in the operations column are carried out and the one with the lowest value (321,856.39) is chosen, with the slack variable h2 as the variable that leaves and its place will be taken by the variable *x3*, see Table 5.

Var	z	x1	x2	x3	h1	h2	h3	Revenue	Operations
z	1	-5.16	0	-5.31	0	0	1.77	0	
h1	0	3	0	3	1	0	-1	1,000,000	1000000/3=333333
h2	0	59.43	0	**60.44**	0	1	-20.41	19,453,000	19453000/60.44=321856
x2	0	-2	1	-2	0	0	1	0	0/-2=0 (se ignora)

Table 5 Middle table
Own Elaboration

Now, at the crossroads of the variables x3 and h2 is the pivot with a value of 60.44, which is why all of that line (R3) is divided by 60.44 to transform it into a value of 1. Once the pivot has been established as 1, we proceed to leave the entire column of x3 in zeros (0) relying on the formulas established in the operations column.

In this case, all the values of the first line are positive or greater than or equal to zero, which is interpreted as the optimal solution with a profit of $1,709,057.41, see Table 6.

Var	z	x1	x2	x3	h1	h2	h3	Revenue	Operations
z	1	0.06	0	0	0	0.087	-0.02	1709057.4	R1+5.31*R3
h1	0	0.05	0	0	1	-0.049	0.01	34430.84	R2-(3*R3)
x3	0	0.98	0	**1**	0	0.016	-0.33	321856.39	R3/60.44
x2	0	-0.033	1	0	0	0.033	0.36	643712.77	R4+(2*R3)

Table 6 Calculated máximum gain.
Our Elaboration

Gratitude

We appreciate both the facilities and the information provided by the gasoline sales service station in Ebano, S.L.P., Mexico.

To the Polytechnic University of Altamira and the Academix Body in Consolidation UPALT- CA6- "Quality and Productivity".

Conclusions

To obtain the máximum profit it's necessary to sell the following quantities in liters of each of the 3 products:

X1 = 34,432 magna gasoline liters X2 = 643,712 premium gasoline liters X3 = 321,856 diesel liters To get a monthly earn of $1,709, 057.41

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Deduction and construction of a discrete probability function (Binomial and Geometric) with analytical development of expected value and variance

Deducción y construcción de una función de probabilidad discreta (Binomial y Geométrica) con desarrollo analítico de valor esperado y varianza

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Abstract

The present work deals with the construction and deduction of a discrete probability function, where the objective is the understanding of the mathematical foundations of probability for its correct application, interpretation and verification in solving probability problems. For this purpose we rely on the theory of didactic situations of Brousseau (1997) and Sadovsky (2005). Teaching strategies are proposed with didactic model materials that improve student learning by promoting the analysis and understanding of various foundations of probability (Panizza, 2003). The importance of close communication between teachers and students in the teaching-learning process for the resolution, interpretation and verification of probability problems is highlighted. The software used in this research was Wolfram Mathematica, which facilitates mathematical writing, calculations, and the construction of graphs using a simple interface. This software makes work easier by fostering student autonomy in the development of skills for analysis and problem solving. We believe that these materials will contribute to the improvement of teaching-learning processes on the fundamentals and concepts of probability.

Discrete probability function, Expected value, Higher education

Resumen

El presente trabajo trata la construcción y deducción de una función de probabilidad discreta, donde el objetivo es la comprensión de los fundamentos matemáticos de la probabilidad para su correcta aplicación, interpretación y comprobación en la resolución de problemas probabilidad. Para este fin nos apoyamos en la teoría de situaciones didácticas de Brousseau (1997) y Sadovsky (2005). Se proponen estrategias de enseñanza con materiales de modelos didácticos que mejoren el aprendizaje del estudiante fomentando el análisis y la comprensión de diversos fundamentos de la probabilidad (Panizza, 2003). Se destaca la importancia de la estrecha comunicación entre profesores y estudiantes en el proceso la enseñanza-aprendizaje para la resolución, interpretación y comprobación de problemas de probabilidad. El software utilizado en esta investigación fue Wolfram Mathematica, mismo que facilita la escritura matemática, los cálculos y la construcción de gráficas utilizando una interfaz sencilla. Este software facilita el trabajo fomentando la autonomía del estudiante en el desarrollo de las habilidades para el análisis y la resolución de problemas. Consideramos que estos materiales permitirán contribuir en la mejora de los procesos de enseñanza-aprendizaje sobre los fundamentos y conceptos de probabilidad.

Función de probabilidad discreta, Valor esperado, Educación superior

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Introduction

In order to build and deduce a discrete probability function, in the binomial case we start from an ancient activity that continues to be carried out today, such as sowing seeds. At the same time, we use definitions and fundamental concepts in order to develop analytically the demonstrations of expected value and variance, through the application of algebraic procedures. This didactic strategy allows the construction and understanding, in a natural, inductive and deductive way, of the concepts involved. The teaching of probability and statistics has presented a great development in recent years due to its increasing application in various fields of science, technology and social and administrative disciplines.

Many institutions and educational centers in the world have dedicated great efforts to design and update curricula and specific materials, to alleviate the difficulties in the teaching-learning process of probability and improve the quality of education.

For this work we initially rely on the Theory of Didactic Situations due to the conceptual nature of probability and its application (Brousseau, 2002) (Brousseau, 1986) (Sadovsky, 2003) (Barreiro and Casseta, 2015) (Pérez y Beltrán, 2011) (Pérez y Pérez, 2018). The theory of didactic situations makes it possible to generate adequate conditions for the analysis of mathematical knowledge, under the hypothesis that they are not built spontaneously (Panizza, 2003). That is, it is based on the premise that knowledge is not transmitted from one person to another automatically, but rather that the individual constructs his or her own knowledge. In this sense, the role of the teacher is very important because a rigorous design and a judicious and appropriate choice of problems depend on him.

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The second approach is the one referring to the constructivist theory that allows developing heuristic strategies so that students choose appropriate methods for solving problems when facing real world situations. This didactic approach stimulates curiosity in students to understand the problem posed. The student thus develops the concepts and procedures to understand and solve problems. Furthermore, students are involved in a permanent process to acquire knowledge in a real context, since, for constructivism, the most important thing is not the new knowledge itself, but to acquire new skills, which allows students to apply what is already known to the understanding of a new problem.

Justification

There are several reasons that justify this work: the first reason is related to the teaching work: it is known that mathematical issues and therefore probability are complex for students of all educational levels, as reported by the literature on Statistical Education (Batanero and Godino, 1997).

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The second reason refers to the importance of probability in school curricula in the educational system in Mexico. For example, at the basic level, students finish their instruction without having assimilated the knowledge and understanding of meanings related to probability.

This partly has to do with the fact that certain topics are often not covered, even if they are within the curricular guidelines, and when they are covered, it is done procedurally and not conceptually. The third reason is that, just as students have difficulties understanding probability concepts, teachers have great difficulty in teaching in a comprehensive and clear way.

Problem Statement

According to the teaching experience and research reports presented in different national and international forums on Statistics and probability, we can affirm that the didactic resources to support teaching are scarce. On the other hand, the textbooks used to teach probability and statistics give more importance to procedure than to understanding; furthermore, the exploratory approach is reduced.

On the other hand, some textbooks such as the following: William Mendenhall (2013), (Mood, Graybill & Boes, 1974); Ronald E. Walpole (2012) only mentions the functions and their respective parameters, but omits detailed demonstrations, and even more so, in the case of George C. Canavos (1988) presents some incomplete developments.

Another important aspect is the scarce national bibliography of probability texts, since they are mainly published in their original language (English, French, German, Russian, Italian, etc.). Books in other languages come from diverse cultural contexts and are focused on the educational systems from which they come. Consequently, these books do not take into consideration the cultural identity of Mexico and its social and educational context. Due to the above, it is considered important to have our textbooks appropriate to the social and educational context of Mexico.

Hypothesis

The selection of didactic strategies that use appropriate examples to promote interest and active participation of students in understanding and solving probability mathematical problems will promote self-learning and the development of skills to understand more complex topics in the classroom.

General objectives

Design didactic strategies for the construction and formulation of analytical demonstrations of the discrete probability function, expected value and variance, through procedures and use of algebraic artifices, identities and differential calculus.

Specific objectives

To have didactic materials that allow the student to strengthen their autonomy and develop their skills and abilities at a higher level. Among them the following can be mentioned:

– Express ideas clearly.
– Structure ideas logically.
– Structure graphs, tables and diagrams that help to obtain the desired result.
– Use appropriate language and mathematical representation.
– Selection of appropriate mathematical tools.
– Demonstrate knowledge and understanding.
– Apply mathematics in different media and contexts.
– Apply problem solving techniques.

Theoretical Frame

Many math problems require proofs. This challenge is a valuable opportunity for the student to apply previous knowledge and develop skills through problem solving. It can be affirmed that the development of these skills is the result of the effort and personal work of each student throughout the teaching-learning process. It is difficult for a student to become a good mathematical problem solver just by passively reading a book. However, the effort on the part of the student to understand and understand a mathematical problem, as well as the use of appropriate techniques can be of great help to solve the problem.

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It is common that, in various mathematical sub-disciplines, it is required to understand key concepts and the relationships between them to solve various mathematical problems.

It is also known that problem solving activities promote the development of skills and abilities in the student. This fact is an argument that justifies the importance of including this approach within the study plans. At the same time, this approach is considered essential in the educational process, since it allows students to understand the scope and usefulness of mathematics in the real world. Throughout life, every individual will encounter problems that must be faced, understood and resolved both inside and outside of school. Therefore, it is considered relevant in developing in the student the ability to solve problems in the classroom.

In a lecture delivered in 1968 George Polya said: "It is well justified that all mathematics texts contain problems. The problems can even be considered as the most essential part of mathematics education" (Pérez and Beltrán, 2011) (Polya, 1965).

Guzmán (1984) comments that "what we should above all provide our students through mathematics is the possibility of acquiring adequate thought habits for solving mathematical and non-mathematical problems. What good can a few theorems and properties related to entities with little meaning serve in your mind if you are going to leave them there hermetically locked up? Problem solving has rightly been called the heart of mathematics, because that is where you can acquire the true value that has brought and attracts mathematicians of all ages. Facing appropriate problematic situations is where motivations, attitudes, habits, ideas for the development of tools can result, being the life of mathematics. When solving problems, you fundamentally learn to understand how our own reasoning works, to master our moods and to increase our self-confidence".

Methods

Examples of probability problems were selected to stimulate the student's interest in analysis and reflection (deep observation of the elements involved and their relationships between them).

From this reading, the reformulation of the problem is proposed to give way to an algebraic development, using it as a guide in the search for ideas for the solution of the problem. In general, this technique leads us at the same time to develop a mathematical process through algebraic tricks that allow a new structuring of the problem and improve its understanding until the desired result is obtained. In short, this is a process of analysis and synthesis.

This didactic strategy has an initial empirical approach in order to solve real world problems. These problems require quantity quantifications and at the same time, they are associated with a qualitative analysis.

Campistrous P. L. and Rizo C. (1996), with great objectivity, point out that traditional procedures are aimed at the actions carried out by teachers in the teaching process where the importance of active student participation is not highlighted. Considering the above, these authors express the following aspects that characterize traditional teaching:

- The stimulation is indirect
- The forms of performance generalized in the student are not achieved, even when they are necessary for life.
- The problems are focused on developing calculus skills and not as a teaching object.
- The difficulty of the problems and their parameters are not very precise, which confuses or makes it impossible to build analogies.
- Particularly, in mathematical analysis problems, the meanings are not adequately worked on.
- Likewise, they present a series of techniques that allow them to face the challenges to solve more complex problems.

Kind of investigation

We can locate this Research in descriptive, characterized by the depth of the mathematical analysis and the development of techniques and strategies to pose and solve problems where the interpretation and verification acquire relevance in its application.

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Theoretical Methods

In these sample problems we address algebraic procedures by applying mathematical artifices to develop proofs of expected value and variance. To process the examples, the Wolfram Mathematica software was used, facilitating the preparation of graphs and calculations. The previous knowledge that is required is knowledge of algebra and differential calculus of higher level.

Development Methodology

Definitions and basic concepts of a discrete random variable:

Expected Value

$$E(X) = \sum_{x=1}^{N} x f_x(x) \quad (1)$$

$$Var(X) = \sum_{x=1}^{N} (X - E(X))^2 f(X) \quad (2)$$

$$Var(X) = E(X^2) - [E(X)]^2 \quad (3)$$

Next, the deduction and construction of a probability function is presented, as well as the analytical demonstrations of expected value and variance respectively, resorting to the application of a meticulous algebraic work supported by artifices and indirect expressions of derivatives and expected value.

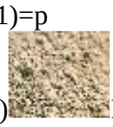
Results

The deduction and construction of the Binomial and Geometric probability functions are presented together with the definitions and proofs of expected value and variance below.

Deduction and construction of the Binomial Probability Distribution Function

4 corn seeds are planted under the same and independent conditions, where the variable X represents the number of seeds that will germinate, the following table shows the possible cases with their respective probabilities.

Coding: 1= success  P(1)=p

0=failure (does not germinate)  P(0)=q

Éxitos X	1 | 2 | 3 | 4	Probabilidad $f(x) = \binom{4}{x} p^x q^{4-x}$
4	1 | 1 | 1 | 1	$P(4) = (1)\, p^4 q^{4-4}$
3	1 | 1 | 1 | 0 1 | 1 | 0 | 1 1 | 0 | 1 | 1 0 | 1 | 1 | 1	$P(3) = (4)\, p^3 q^{4-3}$
2	1 | 1 | 0 | 0 1 | 0 | 1 | 0 1 | 0 | 0 | 1 0 | 1 | 1 | 0 0 | 1 | 0 | 1 0 | 0 | 1 | 1	$P(2) = (6)\, p^2 q^{4-2}$
1	1 | 0 | 0 | 0 0 | 1 | 0 | 0 0 | 0 | 1 | 0 0 | 0 | 0 | 1	$P(1) = (4)\, p^1 q^{4-1}$
0	0 | 0 | 0 | 0	$P(0) = (1)\, p^0 q^{4-0}$

Table 1 Generalizing the problem for n seeds

Éxitos X	1 | 2 | 3 | 4 | ... n	Probabilidad $P(x) = \binom{n}{x} p^x q^{n-x}$
n		$P(n) = \binom{n}{n} p^n q^{n-n}$
n - 1		$P(n-1) = \binom{n}{n-1} p^{n-1} q^{n-(n-1)}$
2		$P(2) = \binom{n}{2} p^2 q^{n-2}$
1		$P(1) = \binom{n}{1} p^1 q^{n-1}$
0		$P(0) = \binom{n}{0} p^0 q^{n-0}$
	Generalizando: $f_x(X) = f_x(X; n, p)$ $= \begin{cases} \binom{n}{x} p^x q^{n-x}, \text{ para } x = 0, 1, 2, .., n \\ 0 \quad \text{en otro caso} \end{cases}$	$= 1$

Table 2

Expected value: $E(X) = \sum_{x=0}^{n} x\binom{n}{x} p^x q^{n-x}$

$= 0\binom{n}{0} p^0 q^{n-0} + 1\binom{n}{1} p^1 q^{n-1} + \ldots n\binom{n}{n} p^n q^{n-n}$

$$\sum_{x=1}^{n} x \frac{n(n-1)!}{x(x-1)!(n-x)!} p^x q^{n-x} p^1 p^{-1} = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

variable change $y = x - 1 \Rightarrow x = y + 1$

$x \to n \qquad y = n - 1$

$x \to 1 \qquad y = 0$

$$np \sum_{y=0}^{n-1} \frac{(n-1)!}{y!(n-1-y)!} p^y q^{n-1-y} = np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y q^{n-1-y}$$

$$= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y q^{n-1-y}$$

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By Newton's Binomial

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} y^k x^{n-k}$$

$$(\mathrm{p}+\mathrm{q})^{\mathrm{n}-1} = \sum_{\mathrm{y}=0}^{\mathrm{n}-1} \binom{\mathrm{n}-1}{\mathrm{y}} \mathrm{p}^{\mathrm{y}} \mathrm{q}^{\mathrm{n}-1-\mathrm{y}} = 1$$

$\therefore \mathrm{E}(\mathrm{X}) = \mathrm{np}$

Variance: $\mathrm{Var}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) - [\mathrm{E}(\mathrm{X})]^2$

$$E[\mathrm{X}(\mathrm{X}-1)] = \sum_{\mathrm{x}=0}^{\mathrm{n}} [\mathrm{X}(\mathrm{X}-1)]\mathrm{f}(\mathrm{x})$$

$$= \sum_{\mathrm{x}=0}^{\mathrm{n}} [\mathrm{X}(\mathrm{X}-1)]\binom{\mathrm{n}}{\mathrm{x}} \mathrm{p}^{\mathrm{x}} \mathrm{q}^{\mathrm{n}-\mathrm{x}}$$

$$= 0\binom{n}{0}p^0q^{n-0} + 1(1-1)\binom{n}{1}p^1q^{n-1} + 2(2-1)\binom{n}{2}p^2q^{n-2} \ldots + n(n-1)\binom{n}{n}p^nq^{n-n}$$

$$= 0 + 0 + \sum_{x=2}^{n} [X(X-1)]\binom{n}{x}p^xq^{n-x}$$

$$= 0\binom{n}{0}p^0q^{n-0} + 1(1-1)\binom{n}{1}p^1q^{n-1} + \sum_{x=2}^{n} [X(X-1)]\binom{n}{x}p^xq^{n-x}$$

$$= 0\binom{n}{0}p^0q^{n-0} + 1(1-1)\binom{n}{1}p^1q^{n-1} + \sum_{x=2}^{n} [X(X-1)]\binom{n}{x}p^xq^{n-x}$$

$$= \sum_{x=2}^{n} [X(X-1)]\frac{n(n-1)!}{X(X-1)(X-2)!(n-X)!}p^xq^{n-x}$$

Variable change $y = x-2, \quad x = y+2$

$x = n \qquad y = n-2$

$x = 2 \qquad y = 0$

$$= n(n-1)\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!}p^xp^2p^{-2}q^{n-x} =$$

$$p^2n(n-1)\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!}p^yq^{n-2-y}$$

By Newton's Binomial

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} y^k x^{n-k}$$

$$\therefore \sum_{y=0}^{n-2} \frac{(n-2)!}{(y)!(n-2-y))!}p^yq^{n-2-y} = (p+q)^{n-2}$$

$$\sum_{y=0}^{n-2} \binom{n-2}{y} q^y p^{n-2-y}$$

$$p+q=1 \therefore (p+q)^{n-2} = \sum_{y=0}^{n-2} \binom{n-2}{y} p^y q^{n-2-y}$$

$$E[X(X-1)] = E[X^2] - E[X] = p^2n(n-1)$$

E[X] = np

$$E[X^2] = p^2n(n-1) + \mathrm{np} \therefore E[X^2]$$

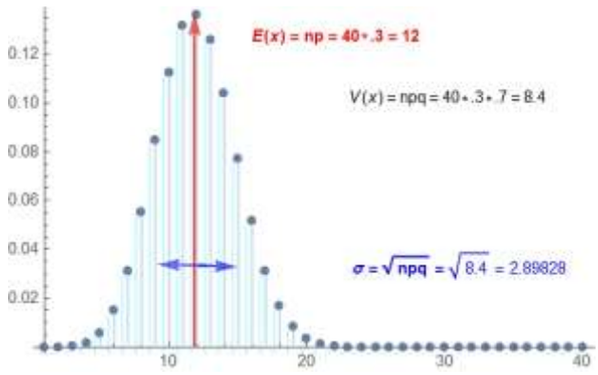
$$= p^2n(n-1) + \mathrm{np} = p^2n^2 - p^2n + \mathrm{np}$$

$$\mathrm{V\,ar}(X) = p^2n^2 - p^2n + \mathrm{np} - (\mathrm{np})^2$$

$$= \mathrm{np}(1-p) = \mathbf{npq}$$

DiscretePlot[

Evaluate@Table[PDF[BinomialDistribution[40, p], k], {p, {0.3}}], {k, 40}, PlotRange -> All, PlotMarkers -> Automatic]



Graph 1

Deduction and construction of the Geometric Probability Distribution Function

We start from a practical application problem: A real estate company sells properties, it is known that a property is sold with a probability V on average. It is known that the sale on one day is independent of the sale on any other day; Find the probability distribution for X, the number of days that there is no sale between two sales

.Sell =**V** Do not sell=**N** $\mathrm{P(V)} = \mathrm{V}$ $\mathrm{P(N)} = \mathrm{N}$

$V + N = 1 \; y \; (1-V) = N, \; 0 < N < 1$

What can happen? Helps define events. In how many ways can each outcome occur? With the help of tables, diagrams, drawings, colors and counting techniques, it will surely be easier to assign probabilities to each result attached to the axioms and theorems of probability.

X → / i	0	1	2	.	n
0	VV				
1	NVV	VNV			
2	NNVV	NVNV	VNNV		
3	NNNVV	NNVNV	NVNNV	.	
.	.	.	.	.	
n	NN . . NVV	NN . . NVNV	NN . . NVNNV	.	VN...NV

Table 3

Assigning probabilities to each event requires having clear concepts and applying the fundamentals of probability.

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Next, the table shows the cases of the number of days of no sale that there are between two days that there are sales

Files	X→ / n↓	X = 0	X = 1	X = 2	.. X = n-1	X = n
1	0	VV				
2	1	NVV	VNV			
3	2	NNVV	NVNV	VNNV		
4	3	NNNVV	NNVNV	NVNNV	...	
. k-1	. n-1	. .	. .	. .	. N .. NVN .. NV	
k	n	NN .. NVV	NN .. NVNV	N..NVNNV	.N .. NVN .. NV	VN..NV
	P (x)	$V^2\sum_{i=0}^{n} N^i$	$V^2\sum_{i=1}^{n} N^i$	$V^2\sum_{i=2}^{n} N^i$	$.V^2\sum_{i=n-1}^{n} N^i$	V^2N^n

Table 4

Notice that in each row of the table there are k non-empty cells, and each cell in the row has two V and (k-1) N letters.

The last row represents the sums of the probabilities of each column. It is worth mentioning that these sums can be rewritten as:

$$\mathbf{V^2\sum_{i=0}^{n} N^i} \quad (4.1)$$

$$\mathbf{V^2\sum_{i=1}^{n} N^i = V^2\sum_{i=0}^{n} N^i - V^2} \quad (4.2)$$

$$\mathbf{V^2\sum_{i=2}^{n} N^i = V^2\sum_{i=0}^{n} N^i - V^2 - N\,V^2} = \mathbf{V^2\sum_{i=1}^{n} N^i - V^2N} \quad (4.3)$$

$$\mathbf{V^2N^n = V^2\sum_{i=0}^{n} N^i - V^2 - N\,V^2 - N^2V^2 - \ldots - N^{n-1}V^2} = \mathbf{V^2\sum_{i=n-1}^{n} N^i - V^2N^{n-1}} \quad (4.4)$$

Fila	X = 0	X = 1	X = 2	..	X = n
1	VV				
2	NVV	VNV			
3	NNVV	NVNV	VNNV		
.	.	.	.	..	
k	N···N VV (k-1)	N···N V N V (k-2) (=1)	.	..	V N···N V (k-1)

Table 5

Solution: Developing the first sum (Table 4) and substituting probability values, respectively: $P(X=0) = V^2\sum_{i=0}^{n} N^i = V^2\sum_{i=0}^{n} N^i = V^2\{N^0 + N^1 + N^2 + \cdots N^{n-1} + N^n\}$ (5)

$$S = \sum_{i=0}^{n} N^i = \{N^0 + N^1 + N^2 + .. N^{n-1} + N^n\} = \{1 + N^1 + N^2 + .. N^{n-1} + N^n\} \quad (6)$$

We multiply (6) by N, we obtain:

$$NS = (N)\{1 + N^1 + N^2 + .. N^{n-1} + N^n\} = \{N^1 + N^2 + N^3 + .. N^n + N^{n+1}\} \mathbf{(7)}$$

Resetting (6) - (7) we get:

$$S = \{1 + N^1 + N^2 + N^3 \ldots\ldots + N^{n-1} + N^n\}$$
$$-NS = -\{N^1 + N^2 + N^3 + \ldots\ldots + N^{n-1} + N^n + N^{n+1}\}$$
$$= 1 - N^{n+1}$$

$$S - NS = 1 - N^{n+1} \Rightarrow S = \frac{1-N^{n+1}}{1-N}, n \to \infty \therefore \lim_{n\to\infty} N^{n+1} = 0$$

$$S = \frac{1}{1-N} = \frac{1}{1-N} = \frac{1}{V} \therefore \lim_{n\to\infty} V^2\sum_{i=0}^{n} N^i \quad (8)$$

$$\lim_{n\to\infty} V^2\sum_{i=0}^{n} N^i = V^2S = V^2\left(\frac{1}{V}\right) = V$$

$$\therefore \mathbf{P(X = 0) = V}$$

Now we can calculate the probabilities for each value of X, according to the last row of table 4.

P (X = 0)	$V^2\sum_{i=0}^{n} N^i = V N^0$
P (X = 1)	$V^2\sum_{i=0}^{n} N^i - V^2 = V - (V^2) = V(1-V) = VN^1$
P (X = 2)	$V^2\sum_{i=1}^{n} N^i - V^2N = V(N) - (V^2)(N) = V(N)(1-V) = VN^2$
P (X = 3)	$V^2\sum_{i=2}^{n} N^i - V^2N^2 = V(N^2) - (V^2)(N^2) = V(N^2)(1-V) = VN^3$
... P (X = n - 1)	..... VN^{n-1}
P (X = n)	$V^2\sum_{i=n-1}^{n} N^i - V^2N^{n-1} = V(N^{n-1}) - (V^2)(N^{n-1}) = V(N^{n-1})(1-V) = VN^n$

Table 6

We can deduce by induction the probability function: $P(X)=VN^i$ which must fulfill:

The sum of all probabilities must be equal to one (that is, the limit when n→∞ of the sum of the last row of table 2 must be equal to one.

$$\sum_{i=0}^{n} P(X) = V^2\sum_{i=0}^{n} N^i + V^2\sum_{i=1}^{n} N^i + V^2\sum_{i=2}^{n} N^i + \ldots + V^2N^n = 1$$

Write: $\sum_{i=0}^{n} P(X) = \lim_{n\to\infty} V^2\sum_{i=0}^{n} (i+1)N^i = 1$ (9)

GARCÍA-NAVA, Elizabeth, HERRERA-MIRANDA, Miguel Apolonio, HERRERA-MIRANDA, Israel and VILLAGÓMEZ-MÉNDEZ, Juan. Deduction and construction of a discrete probability function (Binomial and Geometric) with analytical development of expected value and variance. Journal-Mathematical and Quantitative Methods. 2023

Article **Journal-Mathematical and Quantitative Methods**
June, 2023 Vol.7 No.12 25-35

$V^2 \sum_{i=0}^{n} (i+1)N^i = V^2 \sum_{i=0}^{n} i(N^i) + V^2 \sum_{i=0}^{n} N^i$ (10)

$\lim_{n\to\infty} V^2 \sum_{i=0}^{n} N^i = V$, y que: $(1-V) = N$

$V^2 \sum_{i=0}^{n} i(N^i) = V^2[0N^0 + 1N^1 + 2N^2 + 3N^3 + 4N^4 + \cdots (n-1)N^{n-1} + nN^n]$

$\boldsymbol{W} = \sum_{i=0}^{n} iN^i = 0N^0 + 1N^1 + 2N^2 + 3N^3 + 4N^4 + \cdots (n-1)N^{n-1} + nN^n$ (11)

We multiply (11) by N

$NW = 0N^1N^0 + 1N^1N^1 + 2N^1N^2 + \cdots (n-1)N^1N^{n-1} + nN^1N^n$

$= 0 + N^2 + 2N^3 + 3N^4 + 4N^5 + \cdots (n-1)N^n + nN^{n+1}$(12)

(11) –(12)

$W = 0N^0 + 1N^1 + 2N^2 + 3N^3 + 4N^4 + 5N^5 + \ldots\ldots + (n-1)N^{n-1} + nN^n$

$-NW = -(0 + N^2 + 2N^3 + 3N^4 + 4N^5 + \ldots\ldots + (n-2)N^{n-1} + (n-1)N^n + nN^{n+1})$

$W - NW = N^1 + N^2 + N^3 + N^4 + N^5 + \cdots N^{n-1} + N^n - nN^{n+1}$

$\therefore W - NW = N(1 + N^1 + N^2 + N^3 + N^4 + N^5 + \cdots N^{n-1} - nN^n)$

Write:

$1 + N^1 + N^2 + N^3 + N^4 + N^5 + \cdots N^{n-1} + N^n - (n+1)N^n = \left(\sum_{i=0}^{n} N^i\right) - (n+1)N^n$

$= \sum_{i=0}^{n} N^i = \frac{1}{1-N}$

$W - NW = N\{(\frac{1}{1-N}) - (n+1)N^n)$

Sí, $n \to \infty$: $\lim_{n\to\infty} N^n = 0 \therefore (n+1)N^n = 0$

$W - NW = W(1-N) = \frac{N}{1-N}$

$\Rightarrow W = \frac{N}{(1-N)^2} \therefore W = \sum_{i=0}^{n} iN^i = \frac{N}{(1-N)^2}$

we substitute (10)

$\lim_{n\to\infty}\left\{V^2 \sum_{i=0}^{n} i N^i + V^2 \sum_{i=0}^{n} N^i\right\} =$

$= V^2\left[\frac{N}{(1-N)^2} + \frac{1}{1-N}\right] = (1-N)^2\left[\frac{N+1-N}{(1-N)^2}\right] = \mathbf{1}$(13)

For any value of V

$\lim_{n\to\infty}\{V^2 \sum_{i=0}^{n} i(N^i) + V^2 \sum_{i0}^{n} (N^i)\} = \boldsymbol{N} + \boldsymbol{V} = \mathbf{1}$

Generalizing, given that a probability function must satisfy:

$1.\ f(x) \geq 0, \quad 2\ \sum_{i=0}^{n} f(.\lim_{n\to\infty} x_i) = 1$

Previously we got:

$S = \sum_{i=0}^{n} N^i = \frac{1-N^n}{1-N} \Rightarrow \lim_{n\to\infty} S = \frac{1}{1-N},\ Sí\ |N| < 1$

For this case we know that:

$0 < N < 1,\ V = 1 - N,$ Lo que cumple $|N| < 1$

Expected value:

$E(X) = \sum_{x=0}^{n} xpq^x = (0)pq^0 + 1pq + .. + npq^{n+1}$

$= \sum_{x=1}^{n} xpq^x = \sum_{x=1}^{n} xp(1-p)^x$

$= p\sum_{x=1}^{n} x(1-p)^{x-1+1} = p\sum_{x=1}^{n} x(1-p)^{x-1}(1-p)$

$= p(1-p)\sum_{x=1}^{n} x(1-p)^{x-1}$

Derivative: $\frac{d(1-p)^x}{dp} = (-1)x(1-p)^{x-1}$

$\therefore x(1-p)^{x-1} = -\frac{d}{dp}(1-p)^x$

$[-p(1-p)]\sum_{x=0}^{n} \frac{d}{dp}(1-p)^x$

$= p(1-p)\sum_{x=0}^{n} x(1-p)^{x-1}$

From the series: $\sum_{i=0}^{\infty} k^i$

$\sum_{i=0}^{\infty} k^i = \frac{1}{1-k}$ for: $-1 < k < 1$

$\sum_{i=0}^{n} (1-p)^i = \frac{1}{1-(1-p)} - 1 < (1-p) < 1$

$\frac{1}{p} = (1-p)^0 + \sum_{i=1}^{n} (1-p)^i$

$\frac{1}{p} - 1 = \frac{1-p}{p} = \sum_{i=1}^{n} (1-p)^i$

$\sum_{i=0}^{n} (1-p)^i = 1 + \sum_{i=1}^{n} (1-p)^i$

$\frac{1}{p} = 1 + \sum_{i=1}^{n} (1-p)^i \Rightarrow \sum_{i=1}^{n} (1-p)^i = \frac{1}{p} - 1$

$E(X) = -p(1-p)\frac{d}{dp}(\frac{1-p}{p})$

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$= -p(1-p)\frac{p(-1)-(1-p)1}{p^2} = -p(1-p)(\frac{-1}{p^2})$

$\boldsymbol{E(X) = \frac{p(1-p)}{p^2} = \frac{1-p}{p}}$

$\boldsymbol{E(X) = \frac{q}{p}}$

To obtain Var(X), we resort to:

$E[X(X-1)] = \sum_{x=0}^{n}[X(X-1)]f(X)$

$= \sum_{x=0}^{n}[X(X-1)]\text{pq}^x$

$0(0-1)\text{pq}^0 + 1(1-1)\text{pq} + \sum_{x=2}^{n}[X(X-1)]\text{pq}^x$

$= 0 + 0 + \sum_{x=2}^{n}[X(X-1)]\text{pq}^x$

$p(1-p)^2\sum_{x=2}^{n}[X(X-1)](1-p)^{x-2}$

Derivative: $\frac{d^2}{dp^2}(1-p)^x = x(x-1)(1-p)^{x-2}$

$\therefore E[X(X-1)] = \text{p}(1-p)^2\frac{d^2}{dp^2}\sum_{x=2}^{n}(1-p)^x$

$E[X(X-1)] = p(1-p)^2\sum_{x=2}^{n}[X(X-1)](1-p)^{x-2}$

$= \text{p}(1-p)^2\frac{d^2}{dp^2}\sum_{x=2}^{n}(1-p)^x$

$\sum_{i=0}^{\infty}k^i$

$\sum_{i=0}^{\infty}k^i = \frac{1}{1-k} \quad -1 < k < 1$

for: $-1 < (1-p) < 1$

$\sum_{i=0}^{n}(1-p)^i = \frac{1}{1-(1-p)} = \frac{1}{p}$

$\frac{1}{p} = (1-p)^0 + (1-p)^1 + \sum_{i=2}^{n}(1-p)^i$

$\sum_{i=2}^{n}(1-p)^i = \frac{1}{p} - 2 + p = \frac{1-2p+p^2}{p}$

$\sum_{x=2}^{n}(1-p)^x$

$E[X(X-1)] = p(1-p)^2\frac{d^2}{dp^2}(\frac{1-2p+p^2}{p})$

$= p(1-p)^2\frac{d}{dp}(\frac{p(2p-2)-(1-2p+p^2)}{p^2})$

$= p(1-p)^2\frac{d}{dp}(\frac{2p^2-2p-1+2p-p^2}{p^2})$

$= p(1-p)^2\frac{d}{dp}(\frac{p^2-1}{p^2})$

$= p(1-p)^2\frac{p^2(2p)-(p^2-1)2p}{p^4}$

$= p(1-p)^2\frac{2(p^3-p^3+p)}{p^4} = \frac{2(1-p)^2}{p^2}$

$E(X) = \frac{q}{p} = \frac{1-p}{p}$

$\therefore E[X^2] - E[X] = \frac{2(1-p)^2}{p^2}$

$\Rightarrow E[X^2] = \frac{2(1-p)^2}{p^2} + \frac{1-p}{p}$

$= \frac{2(1-2p+p^2)+p(1-p)}{p^2}$

$= \frac{2-4p+2p^2+p-p^2}{p^2} = \frac{p^2-3p+2}{p^2}$

$E[X^2] = \frac{p^2-3p+2}{p^2}$

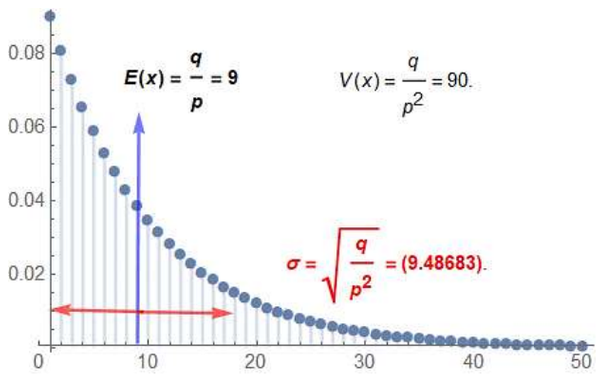
$\boldsymbol{\text{Var}(X) = E(X^2) - [E(X)]^2}$

$= \frac{p^2-3p+2}{p^2} - \frac{(1-p)^2}{p^2}$

$= \frac{p^2-3p+2-1+2p-p^2)}{p^2} = \boldsymbol{\frac{1-p}{p^2}}$

$\boldsymbol{\text{Var}(X) = \frac{1-p}{p^2}}$

DiscretePlot[Evaluate@Table[PDF[Geometric Distribution[p],n],{p,{0.1}}],{n,50},PlotMarkers->Automatic,PlotRange->All]



Graph2

Conclusions

In this work, concepts and proofs have been presented using clear and detailed language sufficient for a higher level student of mathematics. It is possible that these didactic strategies do not allow covering teaching cases in other higher level areas where there are subjects of probability and statistics in the school curriculum. These areas can be Engineering, Biology, Ecology and Sociology. In order for the student to understand the meanings of the fundamental concepts of probability and its procedures, the use of algebraic artifices supported by differential and integral calculus is encouraged.

Other important aspects that were taken into account are:

a) That the student acquire mastery and understanding of the basic concepts of Axiomatic Probability.

b) That students develop the autonomous character to build and develop logical arguments under a demonstration.

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c) It is hoped that students can express themselves correctly using the formal language of Mathematics based on already acquired knowledge.

Suggestions (future work)

To further enrich this didactic approach, it is suggested that the demonstrations be developed by means of the moment generating functions as another way to obtain the expressions of expected value and variance of the main probability distributions.

It is also suggested to use mathematical software such as R, Matlab, Wolfram Mathematica to simulate the behavior of the main probability distributions with examples of real applications. It is suggested to put this proposal into practice to groups of higher level students that allow them the opportunity to understand the conceptual foundations to develop mathematical demonstrations.

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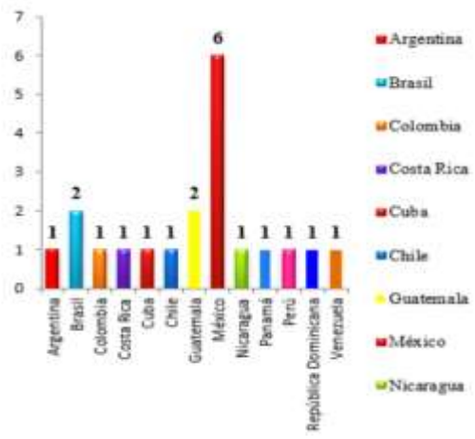

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