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Estimation of DC motor parameters using the least square estimator

Estimación de parámetros de un motor de corriente continua usando el estimador de mínimos cuadrados

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Abstract

The main objective of this work is to present in detail the least squares methodology to estimate the unknown parameters that are part of the mathematical model that represents the behavior of a direct current motor. For this purpose, experimental data of speed and angular position of a direct current (DC) motor with the following characteristics were obtained. DC motor with encoder CN5003-6006, 32 lines of code, 2 output channels: A and B, an approximate weight of 25 g, an operating voltage of 6V/12V (4000/8000 rpm), a current of 30 mA, a shaft diameter of 2mm. The real values of the parameters of the differential equation modeling this motor are

- $R = 0.07\Omega$
- $J = 2.05932971x10^{-6}\ kg * m^2$
- $K_T = 0.002559499\frac{N*m}{A}$
- $K_b = 0.002559499\frac{V}{\frac{Rad}{s}}$

The estimators obtained show that as the size of the sample (experimental data) grows, the value of the estimator tends to be the real value of the unknown parameter, that is, the estimators found comply with the consistency property, which is a requirement. minimum desirable for an estimator. The determination coefficient was used to analyze the convergence of the estimators found to the real value of the unknown parameter.

Parameter estimation, Least squares estimator, Coefficient of determination. DC motor

Resumen

El objetivo principal de este trabajo es presentar la metodología de mínimos cuadrados para estimar los parámetros desconocidos que forman parte del modelo matemático que representa el comportamiento de un motor de corriente continua. Para tal fin, se obtuvieron datos experimentales de velocidad y posición angular de un motor de corriente continua con las siguientes características: Motor de corriente continua con encoder CN5003-6006, 32 líneas de código, 2 canales de salida: A y B, un peso aproximado de 25 g, un voltaje de operación de 6V/12V (4000/8000 rpm), una corriente de 30 mA, un diámetro de flecha de 2mm. Los valores reales de los parámetros de la ecuación diferencial que modela este motor son:

- $R = 0.07\Omega$
- $J = 2.05932971x10^{-6}\ kg * m^2$
- $K_T = 0.002559499\frac{N*m}{A}$
- $K_b = 0.002559499\frac{V}{\frac{Rad}{s}}$

Los estimadores obtenidos muestran que a medida que el tamaño de la muestra (datos experimentales) crece, el valor del estimador tiende a ser el valor real de parámetro desconocido, esto es, los estimadores encontrados cumplen con la propiedad de consistencia lo cual es un requisito mínimo deseable para un estimador. El error cuadrático medio fue utilizado para analizar la convergencia de los estimadores encontrados al valor real del parámetro desconocido.

Estimación de parámetros, Estimador de mínimos cuadrados, Coeficiente de determinación. Motor de corriente continua

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1 Introduction

Engineering is full of models and equations to describe various phenomena (operation of equipment, machines used, etc.). The branch of electrical engineering includes the motor, a machine capable of transforming electrical energy into mechanical energy. This work focuses on the direct current motor, in which the differential equation that models the behaviour of the position and angular velocity is:

$$\frac{d^2\theta}{dt^2} = -\frac{K_b K_T}{RJ}\frac{d\theta}{dt} + \frac{K_T}{RJ}V \qquad (1)$$

where θ is the angular position, J is the rotor moment of inertia, K_b is the counterelectromotive force constant, K_T is the torque constant produced by the magnetic field, R is the resistance of the rotor winding, V is the voltage supplied to the motor, d_θ/d_t is the angular velocity of the motor, and $\frac{d^2\theta}{dt^2}$ is the angular acceleration of the engine.

On many occasions, it is impossible to know the value of one or even more motor parameters, either because they are not specified by the manufacturer or because there is simply no existing information (Amiri, Ibrahim, & Ramli, 2020), which makes it impossible both to model the entire machine and to solve the differential equation. This represents a major problem when implementing control theory in the machine (adaptive control, optimal control) or applying filters on the motor output signals (speed, acceleration) for homogenisation, which is important in applications where high precision is required, such as in motors with a computer numerical control.

Work focused on the formulation of a methodology for estimating the unknown parameters that form part of the differential equation of the DC motor was presented by Zhang (1997) in which he proposes using a conic least squares adjustment as a method for estimating parameters that are unknown in some system. Meanwhile, Escobar-Mejía, Ocampo-Muñoz & Holguín (2008) state that it is possible to get to know totally unknown parameters through the motor plate data, making substitutions in the equations that model the operation of the machine.

Lazarte (2017) performs measurements of the angular velocity ω_n, the armature voltage V_a and the armature current I_a, and based on the data obtained makes use of neural networks for parameter estimation. On the other hand, Amiri, Ibrahim & Ramli (2020) make use of an encoder to take the motor speed measurements and perform the estimation by using a genetic algorithm.

This paper is focused on presenting the least squares methodology to estimate the unknown parameters of a DC motor. It is shown how to estimate one, two, three or all of the unknown parameters that make up the differential equation describing the velocity and angular acceleration of a DC motor, in different cases and combinations.

Section 2 of this work shows the methodology for the estimation of the unknown parameters and how to verify if it is correct. Section 3 presents the results of the implementation of the aforementioned methodology.

2 Methodology

The least squares estimator is a method used to obtain an approximation to a data set (x_i,y_i) by a function $f(x_i)$ such that y_i≈$f(x_i)$ (Sauer, 2013). The method seeks to minimise the squared error between the observed data and the approximation. This error is given by the equation $e:=\sum_{i=1}^{n}\left(y_i - f(x_i)\right)^2$. In this paper the function f is proposed as a linear function, i.e., $f(x) = ax + b$. With the experimental data (x_i, y_i) The least squares method of the DC motor allows us to estimate the constants a and b. For this purpose, in this method the quadratic error e is derived with respect to the variables a and b, and these derivatives are equated to zero to find the minimum values of a and b, respectively.

The method is one of the so-called off-line methods, which means that measurements are first taken of the required data, in this case, the engine speed, and then worked with, without the need for the machine to be in operation.

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Figure 1 shows how the measurements were taken, using a DC motor with an optical encoder, which is responsible for measuring the speed (angular position is taken as θ, angular velocity as θ, angular velocity as θ, angular position as θ, angular velocity as θ). $\frac{d\theta}{dt}$ and the angular acceleration as $\frac{d^2\theta}{dt^2}$). The data were collected by the data acquisition system (DAQ) CompactRIO-9068 belonging to the laboratory of electrical machines and control of the Faculty of Engineering of the Universidad Veracruzana, with a total of 265 samples (i=1, 2 … 265) and saved with the help of LabVIEW software, and then exported for use in MATLAB software for estimation.

The differential equation (1) models the position and angular velocity of the DC motor shown in Figure 2.

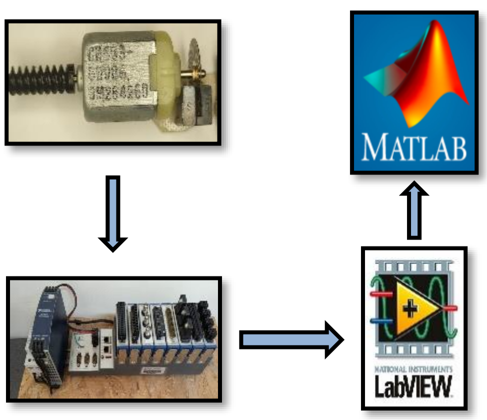

Figure 1 Data acquisition diagram for estimation

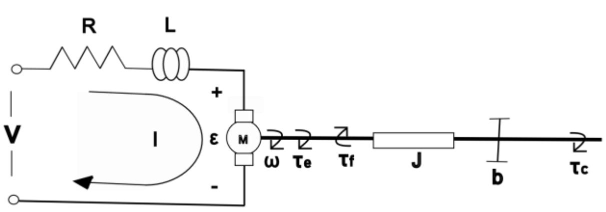

Figura 2 DC motor diagram

Equation (1) can be rewritten in matrix form by making the following changes of variables:

$$x_1 = \theta; \ \dot{x}_1 = x_2 = \frac{d^2\theta}{dt^2} \ ; \ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ ; \ \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{K_b K_T}{RJ} \end{bmatrix} \ ; \ B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{K_T V}{RJ} \end{bmatrix}$$

Consequently, the matrix form of equation (1) is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{K_b K_T}{RJ} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{K_T V}{RJ} \end{bmatrix} \quad (2)$$

$$\dot{x}(t) = Ax(t) + B.$$

If it is assumed that K_T,K_b,R and J are not known, these can be estimated individually or can only be estimated by $-\frac{K_b K_T}{RJ}$ y $\frac{K_T}{RJ}$ (the value of the input voltage is known, since it is the supplied voltage). Suppose that you want to estimate the constants:

$$\alpha := -\frac{K_b K_T}{RJ} \ ; \ \beta := \frac{K_T}{RJ}. \quad (3)$$

Replacing (3) in (2) and performing matrix multiplication, we have:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \alpha x_2 + \beta V \end{bmatrix} \quad (4)$$

where the data for x_1 and x_2 are obtained from taking measurements and $\dot{x}_1$ and $\dot{x}_2$ are obtained from numerically deriving each variable. Since $\dot{x}_1=x_2$ are known data, the estimator is applied to $\dot{x}_2=\alpha x_2+\beta V$. Thus, considering

$$y_i = \dot{x}_{2_i}$$

$$f(x_{2i}) = \alpha x_{2_i} + \beta V$$

The total error is given by

$$e = \sum_{i=1}^{265} \left(\dot{x}_{2_i} - \alpha x_{2_i} - \beta V\right)^2 \quad (5)$$

The method consists of finding the minimum possible error of the selected adjustment, this is done by means of the method for finding the maxima and minima of a function, emphasising that what is sought is the minimum possible error. Therefore, the partial derivatives of e with respect to α and β are calculated and equalled to zero in order to obtain the values of α and β that minimise the error e. This is:

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$$\frac{de}{d\alpha}\left(\Sigma\left(\dot{x}_{2_i}-\alpha x_{2_i}-\beta V\right)^2\right)=0,$$

The above derivative implies that

$$\left(\Sigma x_{2_i}^2\right)\alpha+\left(\Sigma V x_{2_i}\right)\beta=\Sigma \dot{x}_{2_i}x_{2_i} \quad (6)$$

While

$$\frac{de}{d\beta}\left(\Sigma\left(\dot{x}_{2_i}-\alpha x_{2_i}-\beta V\right)^2\right)=0$$

gives what

$$\left(\Sigma x_{2_i}V\right)\alpha+\left(nV^2\right)\beta=\Sigma \dot{x}_{2_i}V \quad (7)$$

As can be seen, with equations (6) and (7) a system of 2x2 linear equations is formed in the variables α and β which, when solved, results in the following values of α and β.

$$\alpha=\frac{\left(\Sigma \dot{x}_{2_i}x_{2_i}\right)\left(nV^2\right)-\left(\mathrm{V}\Sigma \dot{x}_{2_i}\right)\left(\mathrm{V}\Sigma x_{2_i}\right)}{\left(\Sigma x_{2_i}^2\right)\left(nV^2\right)-\left(\mathrm{V}\Sigma x_{2_i}\right)^2} \quad (8)$$

$$\beta=\frac{\left(\Sigma x_{2_i}^2\right)\left(\mathrm{V}\Sigma \dot{x}_{2_i}\right)-\left(\mathrm{V}\Sigma x_{2_i}\right)\left(\Sigma \dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma x_{2_i}^2\right)\left(nV^2\right)-\left(\mathrm{V}\Sigma x_{2_i}\right)^2} \quad (9)$$

Formulas (8) and (9) are the estimators for the unknown constants $\alpha:=-\frac{K_bK_T}{RJ}\ ;\ \beta:=\frac{K_T}{RJ}$.

The least squares methodology can be replicated to estimate one, two or three of the parameters in different combinations. Tables 1 and 2 show the estimators found when K_T or K_b or R or 1/J or K_T/J among other possible combinations are assumed unknown.

Unknown parameters	Estimation α
K_T $\alpha=K_T$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_b^2}{RJ}x_{2_i}^2-\Sigma\frac{K_bV}{RJ}x_{2_i}-\Sigma\frac{K_bV}{RJ}x_{2_i}+n\frac{V^2}{RJ}}$
K_b $\alpha=K_b$	$\alpha=\frac{\Sigma\frac{K_T}{RJ}V\,x_{2_i}-\Sigma\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_T}{RJ}x_{2_i}^2}$
R $\alpha=\frac{1}{R}\ ;\ R=\frac{1}{\alpha}$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_TK_b^2}{J}\,x_{2_i}^2-\Sigma\frac{K_T}{J}K_bV\,x_{2_i}-\Sigma\frac{K_T}{J}K_bV\,x_{2_i}+n\frac{K_TV^2}{J}}$
J $\alpha=\frac{1}{J}\ ;\ J=\frac{1}{\alpha}$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_TK_b^2}{R}\,x_{2_i}^2-\Sigma\frac{K_T}{R}K_bV\,x_{2_i}-\Sigma\frac{K_T}{R}K_bV\,x_{2_i}+n\frac{K_TV^2}{R}}$
K_T,R $\alpha=\frac{K_T}{R}$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_b^2}{J}\,x_{2_i}^2-\Sigma\frac{K_bV}{J}\,x_{2_i}-\Sigma\frac{K_bV}{J}\,x_{2_i}+n\frac{V^2}{J}}$
K_T,J $\alpha=\frac{K_T}{J}$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma\frac{K_b^2}{R}\,x_{2_i}^2-\Sigma\frac{K_bV}{R}\,x_{2_i}-\Sigma\frac{K_bV}{R}\,x_{2_i}+n\frac{V^2}{R}}$
R,J $\alpha=\frac{1}{RJ}\ ;RJ=\frac{1}{\alpha}$	$\alpha=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma(K_b)^2K_T\,x_{2_i}^2-\Sigma K_TK_bV\,x_{2_i}-\Sigma K_TK_bV\,x_{2_i}+n(V)^2K_T}$
K_T,R,J $\alpha=\frac{K_T}{RJ}$	$\alpha=\frac{K_T}{RJ}=\frac{\Sigma V\dot{x}_{2_i}-\Sigma K_b\,x_{2_i}\dot{x}_{2_i}}{\Sigma(K_b)^2\,x_{2_i}^2-\Sigma K_bV\,x_{2_i}-\Sigma K_bV\,x_{2_i}+n(V)^2}$

Table 1 Estimation of the parameter α in different cases

Unknown parameters	Estimation α	Estimation β
K_b,R $\alpha=\frac{K_b}{R}\ ;\ \beta=\frac{1}{R}$	$\alpha=\frac{-\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(n\frac{K_TV^2}{J}\right)-\left(\Sigma V\dot{x}_{2_i}\right)\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)}{\left(\Sigma\left(\frac{K_T}{J}x_{2_i}^2\right)\right)\left(n\frac{K_TV^2}{J}\right)-\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)}$	$\beta=\frac{\left(\Sigma\left(\frac{K_T}{J}x_{2_i}^2\right)\right)\left(\Sigma V\dot{x}_{2_i}\right)-\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma\left(\frac{K_T}{J}x_{2_i}^2\right)\right)\left(n\frac{K_TV^2}{J}\right)-\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)\left(-\Sigma\left(\frac{K_TV}{J}x_{2_i}\right)\right)}$
K_b,J $\alpha=\frac{K_b}{J}\ ;\ \beta=\frac{1}{J}$	$\alpha=\frac{-\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(n\frac{K_TV^2}{R}\right)-\left(\Sigma V\dot{x}_{2_i}\right)\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)}{\left(\Sigma\left(\frac{K_T}{R}x_{2_i}^2\right)\right)\left(n\frac{K_TV^2}{R}\right)-\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)}$	$\beta=\frac{\left(\Sigma\left(\frac{K_T}{R}x_{2_i}^2\right)\right)\left(\Sigma V\dot{x}_{2_i}\right)-\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma\left(\frac{K_T}{R}x_{2_i}^2\right)\right)\left(n\frac{K_TV^2}{R}\right)-\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)\left(-\Sigma\left(\frac{K_TV}{R}x_{2_i}\right)\right)}$
K_T,K_b $\alpha=K_TK_b\ ;\ \beta=K_T$	$\alpha=\frac{\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(n\frac{V^2}{RJ}\right)-\left(\Sigma\dot{x}_{2_i}V\right)\left(-\Sigma\left(\frac{V}{RJ}x_{2_i}\right)\right)}{\left(\Sigma\left(\frac{1}{RJ}x_{2_i}^2\right)\right)\left(n\frac{V^2}{RJ}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{RJ}\right)\right)\left(-\Sigma\left(\frac{V}{RJ}x_{2_i}\right)\right)}$	$\beta=\frac{\left(\Sigma\left(\frac{1}{RJ}x_{2_i}^2\right)\right)\left(\Sigma\dot{x}_{2_i}V\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{RJ}\right)\right)\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma\left(\frac{1}{RJ}x_{2_i}^2\right)\right)\left(n\frac{V^2}{RJ}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{RJ}\right)\right)\left(-\Sigma\left(\frac{V}{RJ}x_{2_i}\right)\right)}$
K_T,K_b,R $\alpha=\frac{K_TK_b}{R}\ ;\ \beta=\frac{K_T}{R}$	$\alpha=\frac{\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(n\frac{V^2}{J}\right)-\left(\Sigma\dot{x}_{2_i}V\right)\left(-\Sigma\left(\frac{V}{J}x_{2_i}\right)\right)}{\left(\Sigma\left(\frac{1}{J}x_{2_i}^2\right)\right)\left(n\frac{V^2}{J}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{J}\right)\right)\left(-\Sigma\left(\frac{V}{J}x_{2_i}\right)\right)}$	$\beta=\frac{\left(\Sigma\left(\frac{1}{J}x_{2_i}^2\right)\right)\left(\Sigma\dot{x}_{2_i}V\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{J}\right)\right)\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma\left(\frac{1}{J}x_{2_i}^2\right)\right)\left(n\frac{V^2}{J}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{J}\right)\right)\left(-\Sigma\left(\frac{V}{J}x_{2_i}\right)\right)}$
K_T,K_b,J $\alpha=\frac{K_TK_b}{J}\ ;\ \beta=\frac{K_T}{J}$	$\alpha=\frac{\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(n\frac{V^2}{R}\right)-\left(\Sigma\dot{x}_{2_i}V\right)\left(-\Sigma\left(\frac{V}{R}x_{2_i}\right)\right)}{\left(\Sigma\left(\frac{1}{R}x_{2_i}^2\right)\right)\left(n\frac{V^2}{R}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{R}\right)\right)\left(-\Sigma\left(\frac{V}{R}x_{2_i}\right)\right)}$	$\beta=\frac{\left(\Sigma\left(\frac{1}{R}x_{2_i}^2\right)\right)\left(\Sigma\dot{x}_{2_i}V\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{R}\right)\right)\left(-\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma\left(\frac{1}{R}x_{2_i}^2\right)\right)\left(n\frac{V^2}{R}\right)-\left(-\left(\Sigma\frac{Vx_{2_i}}{R}\right)\right)\left(-\Sigma\left(\frac{V}{R}x_{2_i}\right)\right)}$
K_b,R,J $\alpha=\frac{K_b}{RJ}\ ;\ \beta=\frac{1}{RJ}$	$\alpha=\frac{\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(\left(n*K_T(V)^2\right)\right)-\left(\Sigma V\dot{x}_{2_i}\right)\left(\left(\Sigma\left(K_TVx_{2_i}\right)\right)\right)}{\left(-\Sigma\left(K_Tx_{2_i}^2\right)\right)\left(\left(n*K_T(V)^2\right)\right)-\left(-\Sigma\left(K_TVx_{2_i}\right)\right)\left(\Sigma\left(K_TVx_{2_i}\right)\right)}$	$\beta=\frac{\left(-\Sigma\left(K_Tx_{2_i}^2\right)\right)\left(\Sigma V\dot{x}_{2_i}\right)-\left(-\Sigma\left(K_TVx_{2_i}\right)\right)\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(-\Sigma\left(K_Tx_{2_i}^2\right)\right)\left(\left(n*K_T(V)^2\right)\right)-\left(-\Sigma\left(K_TVx_{2_i}\right)\right)\left(\Sigma\left(K_TVx_{2_i}\right)\right)}$
K_T,K_b,R,J $\alpha=-\frac{K_bK_T}{RJ}\ ;\ \beta=\frac{K_T}{RJ}$	$\alpha=\frac{\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)\left(nV^2\right)-\left(V\Sigma\dot{x}_{2_i}\right)\left(V\Sigma x_{2_i}\right)}{\left(\Sigma x_{2_i}^2\right)\left(nV^2\right)-\left(V\Sigma x_{2_i}\right)^2}$	$\beta=\frac{\left(\Sigma x_{2_i}^2\right)\left(V\Sigma\dot{x}_{2_i}\right)-\left(V\Sigma x_{2_i}\right)\left(\Sigma\dot{x}_{2_i}x_{2_i}\right)}{\left(\Sigma x_{2_i}^2\right)\left(nV^2\right)-\left(V\Sigma x_{2_i}\right)^2}$

Table 2 Estimation of the parameters α and β in different cases

Collection of experimental data. To corroborate the methodology, the parameters of the mathematical model describing a DC motor with CN5003-6006 encoder were estimated. The characteristics of the motor include 32 lines of code, 2 output channels: A and B, an approximate weight of 25 g, an operating voltage of 6V/12V (4000/8000 rpm), a current of 30 mA, a shaft diameter of 2 mm.

The actual values of the parameters of the differential equation of this motor are:

- $R=0.07\Omega$
- $J=2.05932971x10^{-6}\ kg*m^2$
- $K_T=0.002559499\,\frac{N*m}{A}$
- $K_b=0.002559499\,\frac{V}{\frac{Rad}{s}}$

Considering an applied voltage of 8.5 V, data collection was performed with the data acquisition system (DAQ) CompactRIO-9068 belonging to the Electrical Machines and Control Laboratory of the Faculty of Engineering of the Universidad Veracruzana, shown in Figure 3.

Figure 3 CompactRIO-9068 Data Acquisition (DAQ) System

The experiment consisted of connecting the motor to the DC source and measuring its angular position θ and calculating the angular velocity dθ/dt numerically using the central finite difference method. This process was repeated 20 times, after which the data was averaged. These data were stored and with them the estimation of the parameters in the different cases was carried out.

3 Results

As mentioned in section 2, the least squares methodology was applied to the experimental data obtained from the measurement of the speed of a DC motor to estimate the unknown parameters of the differential equation that models a DC motor.

The results obtained are shown in Tables 3 and 4, in these tables it can be seen that the coefficient of determination R^2 in all cases is greater than 0.99, which shows that the estimated value is very close to the real value due to the fact that the line

$$f(x_{2i}) = -\frac{K_T K_b}{RJ} x_{2_i} + \frac{K_T}{RJ} V \qquad (10)$$

fits the data well $(x_{2_i}, \dot{x}_{2_i})$. Remember that the data x_1 y x_2 are obtained from taking measurements and $\dot{x}_2$ is obtained by numerically deriving each variable x_2.

	Estimation		
	Real	Estimated	R^2
K_T	0.002559499	0.00221497277911894	0.993
K_b	0.002559499	0.00253346667833701	0.996
R	0.07	0.080888095641188	0.993
J	2.05932e-06	2.3796465505e-06	0.993
$\frac{K_T}{R}$	0.0365642714285714	0.0316424682731275	0.993
$\frac{K_T}{J}$	1242.87965524472	1075.57948023725	0.993
RJ	1.441530797e-07	1.6657525853922e-07	0.993
$\frac{K_T}{RJ}$	17755.4236463531	15365.4211462464	0.993

Table 3 Parameter estimation results

In Table 3, the linear function (10) was used assuming as unknown parameters those shown in column 1 of the table. The rows of Table 3 show the result of each case analysed, for example, in row 3 it is assumed that R is unknown, so, for its estimation with (10), what was estimated was a=1/R, that is, the linear function was used.

$$f(x_{2i}) = -\frac{aK_T K_b}{J} x_{2_i} + \frac{aK_T}{J} V$$

Similarly, in row 5, the unknown parameter is assumed to be $\frac{K_T}{R}$ ytherefore, the linear approximation function is

$$f(x_{2i}) = -\frac{aK_b}{J} x_{2_i} + a\frac{1}{J} V$$

where $a = \frac{K_T}{R}$.

Table 4 shows combinations where two, three or four parameters are unknown. It can be seen that the coefficient of determination R2 in all cases is greater than 0.96, which shows that the estimated value is very close to the true value.

	Estimation		
	Real	Estimated	R^2
K_T	0.002559499	0.00221497277911894	0.993
K_b	0.002559499	0.00253346667833701	0.996
R	0.07	0.080888095641188	0.993
J	2.05932e-06	2.3796465505e-06	0.993
$\frac{K_T}{R}$	0.0365642714285714	0.0316424682731275	0.993
$\frac{K_T}{J}$	1242.87965524472	1075.57948023725	0.993
RJ	1.441530797e-07	1.6657525853922e-07	0.993
$\frac{K_T}{RJ}$	17755.4236463531	15365.4211462464	0.993

Table 4 Parameter estimation results

To further corroborate the fit of the unknown parameter estimation, the solution of the differential equation of the engine was found and compared with the experimental data obtained for the angular velocity of the engine. To plot the solution, the unknown parameters were replaced by their estimates. Figure 4 shows the solution of the differential equation with the estimated parameters. $(\alpha := -\frac{K_b K_T}{RJ}\ ;\ \beta := \frac{K_T}{RJ})$ together with the experimental engine speed data. Graphically it can be noted that the solution of the differential equation (1) with the estimated values is close to the experimental speed of the engine analysed, in fact, the coefficient of determination for this case $R^2 = 0.99$, which is synonymous with a good fit.

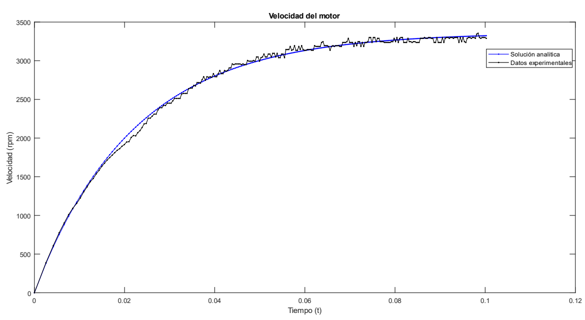

Figure 4 Comparison between experimental data and DC motor speed estimation

5 Conclusions

The objective of estimating the unknown parameters that form part of the mathematical model describing the operation of the DC motor was achieved, since in the various combinations it was possible to find values with errors that do not considerably affect the modelling. Once the unknown parameters have been estimated, it is possible to use the differential equation that models the angular velocity of a DC motor for the required application. It is important to mention that depending on the complexity of the system for which some unknown parameters are required to be estimated, the estimation method must be selected, as in many cases the least squares method may fall short.

6 References

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