# **Numerical and analytical solutions for describing the mechanical response of segment joints for tunnels**

# **Soluciones numéricas y analíticas para describir la respuesta mecánica de juntas entre dovelas para túneles**

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#### **Abstract**

In literature there are several analytical models for describing the mechanical behavior of non-bolted segment joints (longitudinal joints) of shield driven tunnels. These joints modify the structural response of the segmental lining and therefore they affect the global response of this kind of tunnels. The rotational stiffness of these joints is divided into linear and multi-linear models being two typical moment-rotation constitutive equations those exposed by Janssen and Gladwell. The first one considers a linear stress distribution in the longitudinal joint. However, this is not correct according to elasticity theory due to at the edge of the contact area stresses reach infinite values and non-linear stress distributions are produced at the joint. The second typical moment-rotation equation is based on elasticity theory. The analytical results obtained from these moment-rotation equations were compared to numerical results of a 3D model based on Finite Element Method (FEM) using isoparametric solid elements for modeling the concrete segments, whilst for simulating the joint were incorporated contact elements; achieving to confirm that the constitutive equation proposed by Gladwell provides results more precise of the mechanical response of these joints achieving a difference of 0.07% in the maximum capacity of bending moment between the analytical model proposed by Gladwell and 3D numerical model carried out in this study.

En la literatura, existen varios modelos analíticos para describir el comportamiento mecánico de las juntas entre dovelas (juntas longitudinales) no atornilladas de túneles en escudo. Estas juntas modifican la respuesta estructural del revestimiento de dovelas y por lo tanto afectan la respuesta global de este tipo de túneles. La rigidez rotacional de estas juntas se divide en modelos lineales y multi-lineales siendo dos ecuaciones constitutivas típicas momento-rotación las expuestas por Janssen y Gladwell. La primera considera una distribución lineal de esfuerzos en la junta longitudinal. Sin embargo, esto no es correcto de acuerdo con la teoría de elasticidad debido a que en el borde del área de contacto esfuerzos alcanzan valores infinitos y distribuciones de esfuerzos no-lineales se producen en la junta. La segunda ecuación momento-rotación está basada en la teoría de elasticidad. Los resultados analíticos obtenidos de estas ecuaciones momentorotación fueron comparados con resultados numéricos de un modelos 3D basado en el Método del Elemento Finito (MEF) usando elementos sólidos isoparamétricos para modelar las dovelas de concreto, mientras que para modelar la junta fueron incorporados elementos de contacto; logrando confirmar que la ecuación constitutiva propuesta por Gladwell proporciona resultados más precisos de la respuesta mecánica de estas juntas logrando una diferencia de 0.07% en la capacidad máxima de momento flexionante entre el modelo analítico propuesto por Gladwell y el modelo numérico 3D desarrollado en este trabajo.

**Segment joints, Numerical models, Analytical models**

**Juntas entre dovelas, Modelos numéricos, Modelos analíticos**

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#### **Introduction**

Engineering solutions for describing a physical phenomenon can be classified as analytical, experimental, empirical and numerical models. In the case of segmental tunnel engineering, the lining cannot be considered as a continuous structure due to the presence of joints (longitudinal joints or segment joints and circumferential joints or ring joints) as is shown in Figure 1. In this figure is possible to observe the basic parts of a segmental tunnel lining, showing three segmental rings.



**Figure 1** Basic parts of a segmental tunnel lining *Source: Own elaboration*

These joints are formed due to constructive process of the shield driven tunnels given by Tunnel Boring Machine (TBM). The longitudinal joints are in-between the segments in a ring, also called segment joints, whilst circumferential joints are in-between the adjoining rings. The segment joints provide flexibility at the lining, which modifies its structural response, the contact area developed between segments might have a reduced thickness compared to the segment thickness (Blom, 2002) (Figure 2).



**Figure 2** Details of a non-bolted segment joint (longitudinal joint) *Source: Own elaboration*

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These joints transfer bending moments and normal force by contact (Figure 2), and these are unable to transfer tensile forces since the segments are not physically connected (nonbolted joints).

The segment joint acts as a concrete hinge when adjoining segments are relatively rotating to each other. This concrete hinge has resistance against the rotation and bending moments occur (Blom, 2002). The rotational stiffness of the segment joint depends on the contact area, the normal force *N* and the rotation itself.

#### **Analytical models for segment joints**

The segment joints have been studied analytically for several authors (Lee and Ge, 2001; Lee *et al*., 2001; Blom, 2002; Blom, 2003; BTS, 2004; Hefny *et al*., 2004, Li *et al*., 2015) developing or describing analytical models for representing, by means of constitutive momentrotation equations, the mechanical behavior of non-bolted segment joints of shield driven tunnels.

For example, Equation 1 depicts the constitutive moment-rotation equation proposed by Janssen (van der Vliet, 2006), this equation was obtained from a linear stress distribution at the joint. Also, this considers that the joint is not able to develop tensile stresses and it proposes a linear branch and a non-linear branch.

$$
\phi = \begin{cases} 12 \frac{M}{Ebh^2} & \text{if } M < \frac{1}{6}Nh \\ \frac{8N}{9Ebh \left(\frac{2M}{Nh} - 1\right)^2} & \text{if } M \ge \frac{1}{6}Nh \end{cases} \tag{1}
$$

Where *h* is thickness of joint, *b* is width of joint, *E* is modulus of elasticity of material. *N, M* and  $\phi$  are normal force, bending moment and flexural rotation present at the joint, respectively. Whilst Equation 2 depicts the behavior proposed by Gladwell (van der Vliet, 2006), whose proposal is based on the theory of elasticity. Similar to Janssen, Gladwell proposes an analytical model with a linear part and nonlinear part.

$$
\mu = \begin{cases} \frac{\pi}{16(1-\nu^2)} \Phi & \text{if } \Phi \le \frac{8(1-\nu^2)}{\pi} \\ 1 - \frac{4(1-\nu^2)}{\pi} \frac{1}{\phi} & \text{if } \Phi > \frac{8(1-\nu^2)}{\pi} \end{cases}
$$
(2)

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Where:

$$
\mu = \frac{M}{1/2Nh}; \quad \phi = \phi \frac{hbE}{N}
$$

Being  $\mu$  dimensionless formulation in function of moment for the moment-rotation equation,  $\Phi$  dimensionless formulation in function of rotation for the moment-rotation equation,  $h$  thickness of joint,  $b$  width of joint,  $\nu$ Poisson ratio of material and *E* modulus of elasticity of material.  $N$ ,  $M$  and  $\phi$  are normal force, bending moment and flexural rotation present at the joint, respectively. There are three main differences between these two analytical models:

- The initial rotational stiffness obtained from Gladwell model is higher than that determined using Janssen's model.
- According to Gladwell's model, the joint requires a higher bending moment to open.
- The non-linear branch proposed in Gladwell's model more rapidly reaches the maximum moment capacity.

## **Numerical models for segment joints**

Numerical models are performed to solve mathematical problems using a combination of a "*large"* number of mathematical equations to find an approximate solution to different physical problems. In the case of segment joints, in addition to developing analytical models, there are different numerical models to represent their mechanical behavior. Correct modeling of the interaction between segments (segment joint) is vital for representing in a realistic way the structural behavior of a segmental tunnel.

There are several numerical models (Luttikholt, 2007; Vervuurt, 2007) to represent a segmental tunnel lining that use beam elements to simulate the concrete segments, whilst the longitudinal joints can be modeled by using linear or non-linear rotational springs defined by some analytical model and the behavior of these springs is implemented in each joint to model, together with beam elements, a segmental ring. Some numerical models include plane or shell elements for modeling the concrete segments, whilst the joints are represented by means of contact or interface elements.

Other models more detailed considering solid elements for representing the concrete segments and the joints are simulated by using contact elements that update the stiffness of the joint in each load step.

### **Strategy of numerical modeling**

The strategy of numerical modeling included 3D solid isoparametric elements for representing the concrete segments considering an elastic-linear behavior (Figure 3). In addition, 2D contact elements were used to simulate the mechanical response of segment joint (Figure 3), updating its stiffness during the loading process, achieving that the structural analysis be more precise under a reasonable computational time and effort. The segments were modeled by using an 8-node solid element identified in ANSYS (2016) as SOLID65, whilst the joint was considered by means of the contact elements identified as CONTA173 and TARGE170. The contact between the surfaces was considered as perfectly rough corresponding an infinite friction coefficient, achieving only rotations in the joint. On the other hand, 2D contact elements were used for representing a rigid surface by means of slave and master nodes (Figure 3) for applying the normal force and the bending moment as it mentioned in the analytical models exposed in a previous section. However, due to in the nonlinear branch of moment-rotation relation there is a part where under "small" increments of bending moment, rotations increase rapidly, in the numerical model were applied rotations and not bending moments.



**Figure 3** Numerical model of a segment joint by using 2D contact elements and 3D solid elements *Source: Own elaboration*

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Table 1 depicts the elastic mechanical properties of the material used in the analytical and numerical models to represent the mechanical behavior of the segment joint. The dimensions of the joint used for this study also are shown in Table 1. The normal force *N* applied to the joint was of 245 kN.



**Table 1** Mechanical properties of concrete, dimensions of segment joint and normal force used in the analytical and numerical models

### **Numerical and analytical results**

The results of analytical and 3D numerical models are depicted in Figure 4, where it is possible to observe that the solution obtained from analytical model proposed by Gladwell is more similar to the obtained from 3D numerical model than the achieved for analytical model proposed by Janssen. The numerical model presents a maximum capacity of bending moment of 22.05 kN-m, whilst in the constitutive equations proposed by Gladwell and Janssen, bending moments of 22.03 kN-m and 21.58 kN-m are obtained, respectively.

The 3D numerical model presents a difference of approximately 0.07% of bending moment capacity obtained from Gladwell's model.



**Figure 4** Comparison between analytical and numerical models for representing the mechanical behavior of the segment joint *Source: Own elaboration*

The joint presents a linear behavior until a rotation of approximately 0.0002 rad, until this point, the joint is closed. Then, a non-linear behavior caused for the opening of the joint is presented achieving a maximum capacity of bending moment of 22.05 kN-m and an ultimate rotation of 0.04 rad. This value of rotation corresponds the value applied to numerical model.



**Figure 5** Contact pressures obtained from 3D numerical model: a) linear behavior (rotation of 0.0002); b) nonlinear behavior (ultimate rotation of 0.04 rad) *Source: Own elaboration*

In Figure 5 the contact pressures obtained from numerical model are depicted, including the corresponding values to rotation of 0.0002 rad. and the ultimate rotation applied to the model (0.04 rad.), achieving a maximum pressure contact of 68.0 MPa.

#### **Conclusions**

The analytical solutions described in this study and the numerical solution obtained from a 3D model based on Finite Element Method (FEM) carried out to represent a typical non-bolted segment joint, indicate that the constitutive moment-rotation equation proposed by Gladwell provides results more precise about its mechanical behavior, achieving a difference of 0.07% in the maximum capacity of bending moment between the analytical model proposed by Gladwell and 3D numerical model performed in this study, considering solid elements for modeling the segments and contact elements for representing the joint.

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