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Non-homogeneity of energy density in a vertical wind profile for open-air laboratory tests

No-homogeneidad de densidad de energía en un perfil vertical de viento para pruebas de laboratorio a cielo abierto

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Abstract

The research presents the calculation by numerical methods of the energy density for a typical vertical wind profile, which is obtained from data acquired by the Regional Center for Wind Technology (CERTE) in the region of Juchitán de Zaragoza, Oaxaca, Mexico. The vertical wind profile considered presents inhomogeneities in energy density and speed, unlike the controlled conditions in a laboratory with a wind tunnel. The energy density allows us to calculate correction factors on the measured parameters of a wind turbine when it is tested under free wind laboratory conditions. The correction factor for the power coefficient in a small wind turbine with a capacity to generate 30 kW of power, developed by CIATEQ AC Centro de Tecnología Avanzada, hypothetically measured under free wind conditions, is also shown. These correction factors can also be useful in estimating the power generated when laboratory conditions are not available.

Energy density, Inhomogeneity, Numeric method

Resumen

La investigación presenta el cálculo por métodos numéricos de la densidad de energía para un perfil vertical de viento típico, que se obtiene a partir de datos adquiridos por el Centro Regional de Tecnología Eólica (CERTE) en la región de Juchitán de Zaragoza, Oaxaca México. El perfil vertical de viento considerado presenta no homogeneidades en la densidad de energía y en la velocidad a diferencia de las condiciones controladas en un laboratorio con un túnel de viento. La densidad de energía nos permite calcular factores de corrección en los parámetros medidos de un aerogenerador cuando se prueba en condiciones de un laboratorio a viento libre. También se muestra el factor de corrección para el coeficiente de potencia en un aerogenerador pequeño con capacidad de generar 30 kW de potencia, desarrollado por CIATEQ A.C. Centro de Tecnología Avanzada, hipotéticamente medido bajo condiciones de viento libre. Estos factores de corrección pueden ser útiles también en la estimación de la potencia generada cuando no se tienen condiciones de laboratorio.

Densidad de energía, Inhomogeneidad, Método numérico

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Introduction

The consumption of electrical energy is increasing due to the growth in the number of users, whose demand needs were more limited than today. In contrast to a few years ago, the needs have increased due to information technologies in conjunction with the emergence of Industry 4.0.

Electricity providers see wind energy as an important opportunity to take advantage of a renewable resource. In MEXICO in particular, electricity generation from wind energy in the state of Oaxaca is 2,749 MW Apud (Alberto, 2022). This is the site of the Centro Regional de Tecnología Eólica (CERTE), a 32-hectare facility located in the vicinity of the village of La Ventosa, municipality of Juchitán, Oaxaca Apud (INNEL, 2022). CERTE supports wind turbine manufacturers and suppliers in the testing and technological improvement of their products under local conditions, has the characteristic of being exposed to intense winds, thus creating the most important wind tunnel in the region, and can therefore be considered a class I site according to the IEC 61400-1 standard.

In order to measure and evaluate the quality characteristics of wind turbines, large and expensive laboratories are needed. One of these is the wind turbine laboratory of the National Renewable Energy Centre (CENER) in Pamplona, SPAIN, whose infrastructure is dedicated to testing complete wind turbines and their individual components according to international Apud standards (CENER, 2022).

Similarly, the Centro de investigaciones Energéticas, Medioambientales y Tecnológicas (CIEMAT) in Madrid, SPAIN, has a laboratory where power curve, duration, acoustic noise, operation and safety Apud tests can be performed (CIEMAT, 2022).

The creation of this type of laboratory requires a lot of resources, which is why small wind tunnels are chosen and tests are carried out on scale models using similarity techniques. Similarity techniques are well known and accepted, offering a resource-saving solution.

Tests and trials at real scales are more free of possible errors, so a large capacity laboratory is preferred to simulate real conditions on the operation of wind turbines.

With the calculations proposed in this work, the feasibility of implementing a free wind laboratory that allows to reproduce the measurements of a wind tunnel will be determined.

In order to achieve this, chapter 2 presents the vertical wind profiles that we find in practice, described according to the IEC-61400-2 standard. Likewise, the energy conservation equation is used to obtain the energy density distribution for the wind profile considered.

In chapter 3 we present the numerical method used in the calculation of the energy density function. We also present the calculation of the effective speed for a small wind turbine and the correction factor in the power coefficient for measurements in a laboratory in free wind.

2. Metodology

2.1. Design considerations

According to the IEC61400-2 Standard, the tests to verify the design data of a wind turbine are power, rotational speed, shaft torque and maximum design rotational speed. For engineering studies it is proposed to use the empirical formulas of the exponential model and the logarithmic model, both based on the Monin-Obukhov similarity theory, which dates back to 1954 Apud (Yu Cheng, MIngming Zhang, Ziliang Zhang, Jianzhong Xu, 2019).

The laboratories perform static and dynamic blade testing (IEC TS-61400-23), mechanical and electrical power train, power curve (IEC61400-12-1), acoustic noise (IEC 61400-11), power quality (IEC 61400-13) and mechanical loads (IEC 61400-21).

IEC61400-12-1 defines in point 6.2 that the wind speed to be measured is the average magnitude of the horizontal component of the velocity vector which includes only the longitudinal and lateral turbulence components but not the vertical component. The wind speed measurement shall be made with an anemometer which shall be calibrated before and after the field measurement. The difference between the calibration regression lines shall be between 0.1 m/s in the range of 6 to 12 m/s. The temperature sensor and the humidity sensor shall be mounted outside 10 m from the height of the rotor centre.

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The uncertainty in wind speed measurement combines several sources of uncertainties, such as flow distortion (turbulence), mounting and anemometer characteristics. In Annex F. 2 "Wind tunnel requirements" mentions that the wind tunnel shall be equipped to carry out accurate anemometer calibration operations.

IEC 61400-22 deals with certification and conformity testing for wind turbines in each of their components according to IEC 61400-1, 61400-2, 61400-3 61400-23 and ISO/IEC 17021, the latter for laboratories.

In particular the IEC 61400-2 standard mentions the measurement of the power curve, for which it is required to measure the kinetic energy carried by the wind entering the wind turbine. In a wind tunnel the velocity profile is constant and its value is accurately determined with the help of an anemometer, and therefore the kinetic energy can be calculated. However, in free wind conditions, the velocity profile is not constant, as well as temperature variations. In this case, the energy density as a function of height can be obtained from the velocity profile and the conservation equation. This can help to obtain the power curve.

There is an important factor caused in a wind tunnel, known as tunnel interference, for which a correction factor is required. This effect is caused by the walls influencing the actual flow velocity entering the rotor of Apud wind turbines (Jaeha Ryi, Wook Rhee, Ui Chang Hwang, Jong-Soo Choi, 2014). This effect is not present in free wind measurements.

The wind profile available for this work is that found in the lower part of the Atmospheric Boundary Layer (ABL) which is known as the Atmospheric Surface Layer (ASL) of maximum variable height between 60 and 100 meters. Wind speed tends to increase with height, as the earth's surface exerts a frictional action that causes the wind speed to be zero at the surface, rougher surfaces will have more influence on the height of the atmospheric layer than smoother surfaces Apud (J.F. Manwell, J.G.Mcgowan, A.L. Rogers, 2009).

2.2 Theoretical model

Wind profiles below the atmospheric layer are modelled through an exponential function given by equation (1) and/or a logarithmic function given by equation (2), according to IEC 61400-2.

Exponential formula of the vertical wind profile.

$$V_z = V_{ref}\left(\frac{Z}{Z_{ref}}\right)^{\propto} \quad (1)$$

Donde

V_z : Wind speed to be estimated at a height Z above ground level .
V_{ref} : Reference speed generally at a height of between 2 and 10 m.
α : roughness exponent, for our study it is 0.2.

Logarithmic formula of the vertical wind profile.

$$V_z = V_{ref}\frac{ln\left(\frac{Z}{Z_O}\right)}{ln\left(\frac{Z_{ref}}{Z_O}\right)} \quad (2)$$

Where:
Z_o : Roughness length, for our study is 0.25 m.

The conservation of energy equation for an open system is given by

$$\frac{d}{dt}e_v(\vec{r},t) + \vec{\nabla}\cdot e_v(\vec{r},t)\vec{u}(\vec{r},t) = \vec{\nabla}\cdot \tilde{\sigma}\vec{u}(\vec{r},t) + f_v\cdot\vec{u}(\vec{r},t) + \vec{\nabla}\cdot\vec{q} \quad (3)$$

Where $e_v\left(\vec{r},t\right)$ is the total energy density of the system (kinetic energy, potential energy and internal energy), $\vec{u}\left(\vec{r},t\right)$ is the vector is the velocity field, σ is the stress tensor, f_v are the external forces (gravity, electromagnetic fields, etc.) and is the heat flux.

For a steady state it has to be

1. $\frac{d}{dt}e_v\left(\vec{r},t\right)=0$ The change in energy does not depend on t.

And by making the following approximations:

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2. $\overrightarrow{f_g} \cdot \vec{u}\left(\vec{r},t\right) = 0$ Considering only gravity as an external force and the same perpendicular to wind speed.

3. $\vec{\nabla} \cdot \vec{q} = 0$ Disregarding any heat transfer (it is very small).

Equation (3) reduces to

$$\vec{\nabla} \cdot e_v(\vec{r},t)\vec{u}(\vec{r},t) = \vec{\nabla} \cdot \tilde{\sigma}\vec{u}(\vec{r},t) \tag{4}$$

For the hydrostatic case we have that by separating the stress tensor into pure normal and pure shear components we can rewrite equation (4) as

$$\vec{\nabla} \cdot e_v(\vec{r},t)\vec{u}(\vec{r},t) = \vec{\nabla} \cdot (p\tilde{I} + \tilde{\tau})\vec{u}(\vec{r},t) \tag{5}$$

Or rearranging terms

$$\vec{\nabla} \cdot \left[\rho\left(\frac{1}{2}\vec{u}(\vec{r},t) \cdot \vec{u}(\vec{r},t) + u_m' + \frac{p}{\rho}\vec{u}(\vec{r},t)\right)\right]_v = \vec{\nabla} \cdot \tilde{\tau}\vec{u}(\vec{r},t) \tag{6}$$

Where is the pressure and is the shear stress field. We define a function as

$$f_k(x,y,z) = \rho\left(\frac{1}{2}\vec{u}(\vec{r},t) \cdot \vec{u}(\vec{r},t) + u_m' + \frac{p}{\rho}\vec{u}(\vec{r},t)\right) \text{n} \tag{7}$$

Which is the energy density plus a pressure term, i.e. an energy density with enthalpy. Substituting into equation (6) and taking only the z-component we have

$$\frac{\partial}{\partial z} f_k(x,y,z)u_z(y) = \frac{\partial}{\partial y}\tau_{yz}u_z(y) \tag{8}$$

Finally, for a Newtonian fluid we have

$$\left(\frac{\partial^2}{\partial y^2}u_z(y)\right)u_z(y) + \left(\frac{\partial}{\partial y}u_z(y)\right)^2 - \frac{g(y)u_z(y)}{\mu} = 0 \tag{9}$$

Where

$$g(x,y,z) = \frac{\partial}{\partial z} f_k(x,y,z) \tag{10}$$

And

$$\tau_{yz} = \mu\frac{\partial}{\partial y}\left(u_z(y)\right) \tag{11}$$

Discretising equation (9) for constant meshing gives the numerical model

$$\left(\frac{u_z(y_{i+2}) - 2u_z(y_{i+1}) + u_z(y_i)}{h^2}\right)u_z(y_i) + \left(\frac{u_z(y_{i+1}) - u_z(y_i)}{h}\right)^2 - \frac{g(y)u_z(y_i)}{\mu} = 0 \tag{12}$$

With the help of equation (12), giving values for the velocity field, it can be solved iteratively to obtain the derivative of the energy density, i.e. the function.

The power coefficient is the main parameter to obtain the mechanical power of the wind turbine. The amount of power that can be extracted from the air by a wind turbine depends on the maximum value of the Cp known as Betz limit, Apud (Thomas, 2009).

The power coefficient is given by

$$Cp = \frac{P}{\frac{1}{2}\rho A u_z^3} = \frac{P}{Au\left(\frac{1}{2}\rho u_z^2\right)} \tag{13}$$

Where P is the electrical power generated, is the density and A is the swept area of the wind turbine.

The denominator of equation (13) is the kinetic energy contributed by the wind, however, according to equation (7) the total energy contributed by the wind flow includes the enthalpy, so an improvement to the definition of the power coefficient would be as follows

$$Cp = \frac{P}{Au\left(\frac{1}{2}\rho u_z^2 + h\right)} \tag{14}$$

Where h is the enthalpy contained in the wind.

For a variable wind profile the velocity used in equation (14) will be the average value given by

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$$u_z = \frac{1}{b-a}\int_a^b u_z(y)dy \qquad (15)$$

Where a is the minimum height and b is the maximum height of the swept area of the wind turbine.

Results

A free wind laboratory in Mexican territory has to be adapted to the winds of MEXICO. Typical wind data for the region of Juchitan de Zaragoza, Oaxaca, are available from CERTE, which publishes measured values of pressure, temperature, wind speed and wind direction at different heights throughout the year, these data are available at http:/aems.ineel.mx/. The following values shown in table 1 were taken from this site, with the permission of CERTE.

Heights in metres m				Statistical data
80	60	40	20	
10.62	10.33	9.98	8.85	Mean
8.35	8.04	8.04	7.34	Median
5.36	4.92	4.58	3.88	Standard deviation
28.77	24.17	21.05	15.06	Sample variance
18.37	17.36	16.67	14.16	Range
4.94	4.83	4.68	4.41	Minimum value
23.30	22.19	21.36	18.57	Maximum value

Table 1 Statistical data on wind speed (m/s) for the month of March 2021 (CERTE)

Figure 2 shows the wind graph for 20, 40, 60 and 80 m heights. It shows the behaviour of the wind speed during the 31 days of the month.

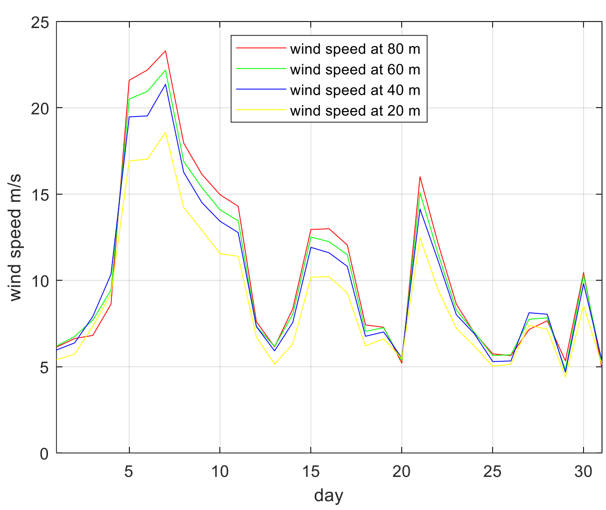

Figure 1 Wind behaviour at different heights for March 2021, in the region of Juchitán de Zaragoza, Oaxaca

The average wind speed at a height of 20 m is 8.85 m/s, which would be our hypothetical working height.

The above experimental wind speed data is used to model the vertical wind profile from equation (12) and obtain a similar wind speed profile and calculate the energy density function. The experimental data were completed by interpolation at different heights using the logarithmic model of equation (2). Figure 3 shows the velocity profile as a function of height from the CERTE measurements, the logarithmic model of the vertical wind profile and the velocities obtained with equation (12).

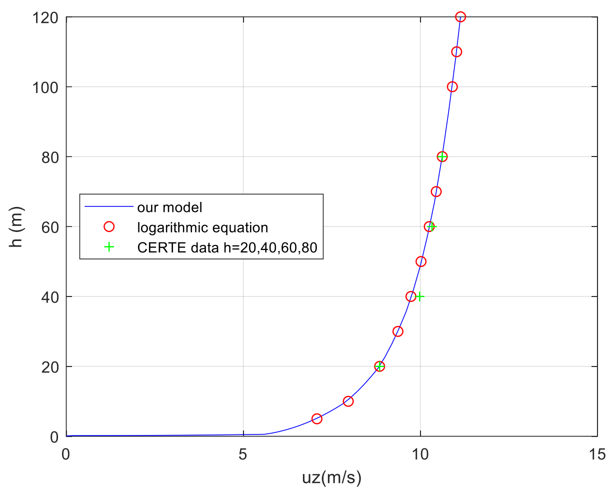

Figure 2 Vertical wind speed profile March 2021

Equation (12) is solved by giving values to the function g(y) (equation (10)) such that substituting them into equation (12) reproduces the experimental velocity profile. For the velocity profile shown in figure (3) the values of the function g(y) are as shown in figure (4). The values found were fitted with an exponential model, which is shown in equation (16) below.

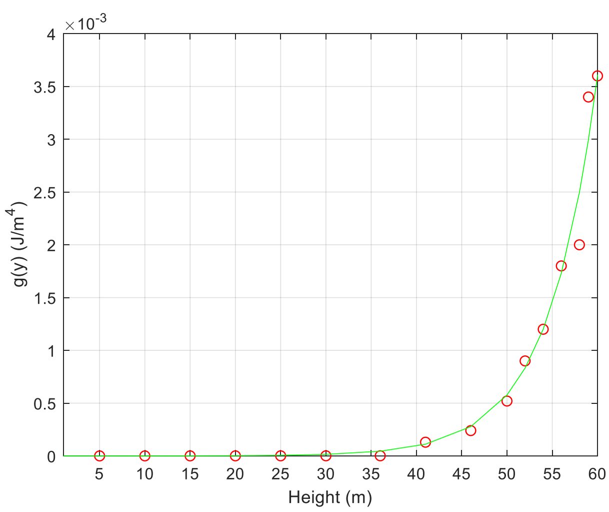

Figure 3 Plot of the derivative of the energy density function g(y)

$$g(y) = 6.097 \times 10^{-8} e^{0.1831y} \quad \left({}^{J}/_{m^4}\right) \qquad (16)$$

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For the enthalpy calculations we will consider the dimensions of a small wind turbine whose specifications are in table 2, such a wind turbine is a development of the Centro de Tecnología Avanzada (CIATEQ A.C.).

Axis of rotation	Horizontal
Rotor orientation	Windward
Class	III-S
Number of blades	3
Rotor diameter	12 m.
Hub height	20.5
Performance Coefficient	0.48
Electrical power rating	30 kW
Rated wind speed	10.5 m/s
Average annual speed	7.5 m/s
Swept Area	113 m2
Direction of rotation	Timetable

Table 2 General characteristics of the wind turbine developed by CIATEQ A. C.

The height at the centre of the wind turbine axis is 20.5 m, the minimum and maximum height of the blades are 14.5 m and 26.5 m respectively. Substituting the heights in equation (10) we have

$$g(26) - g(14.5) = \rho\left[\frac{1}{2}\left(u_z^2(26.5) - u_z^2(14.5) + h(26.5) - h(14.5)\right)\right] \quad (17)$$

Obtaining

$$h(26.5) - h(14.5) = -7.04 \quad \left({}^{J}/_{kg}\right) \quad (18)$$

The difference of the kinetic energy density contained in the wind flow between a height of 14.5m and 26.5m is 38.80 $\left({}^{J}/_{kg}\right)$ while the difference in enthalpy is of the order of 7,039$\left({}^{J}/_{kg}\right)$, This gives an inhomogeneity of the wind profile of 18.64%, which is far from the controlled conditions of a test laboratory. Considering only the wind speed we would have an inhomogeneity of 9%.

If one wanted to test a wind turbine with this wind profile, corrections would have to be made. For example, to measure the power coefficient, a constant speed in the swept area is required. For the following calculations, we will consider the geometry of a small wind turbine whose specifications are listed in table 2.

If we consider a speed under controlled conditions of 8.85 m/s, the density of the air of $\rho = 1.18 \ {}^{kg}/_{m^3}$ and the swept area of 113 m2, the power coefficient is $Cp = \left(2.1639\times10^{-5}\right)P$, where P is the electrical power generated. This value will change if the measurement is made with the wind profile described above, for example, a value close to the real value would be obtained if the average spatial velocity given in equation (15) between the wind turbine heights of 8.5m to 32.5m is used in the calculation of the power coefficient, resulting in an average velocity of 8.72 m/s. With this velocity, same swept area and air density the power coefficient results in $Cp = \left(2.2625\times10^{-5}\right)P$. To have a closer approximation to the real one, we will include the enthalpy given in equation (14) to the kinetic energy, resulting in the power coefficient with a new value of $Cp = \left(1.8939\times10^{-5}\right)P$.

The results show that with the vertical wind profile and the function g(y) we will have a power coefficient closer to the real value. The error considering the velocity at the centre of the axis and the spatial average velocity is 4.56%, considering the enthalpy the error increases to 12.48%. This error can be corrected by finding a correction factor that for this numerical case is equal to 0.95, in such a way that if we multiply the power coefficient obtained in free wind by the correction factor we will obtain the power coefficient for a constant velocity profile, i.e. how it would be if it had been measured in a wind tunnel.

This shows that free wind measurements can be corrected to obtain the results under controlled conditions.

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Conclusions

In this research a value of 0.63 was obtained by means of the standard power coefficient equation and by means of the proposed theoretical methodology the value of the power coefficient of 0.54 considering a power of 30 kW, then the maximum value according to Betz's law is 0.59 fulfilling the proposed theoretical methodology with Betz's Limit.

The hypothesis of constructing a new theoretical model considering the enthalpy is fulfilled since the comparison of the curves; theoretical model, experimental data from CERTE are similar.

The proposed methodology where the new enthalpy variable is considered was validated by means of the logarithmic vertical wind profile for the month of March 2021. The actual wind profile was obtained from data provided by the Centro Regional de Tecnología Eólica (CERTE), for the heights of 20, 40, 60 and 80 m where their anemometers are located. The calculated energy density contains not only the kinetic energy of the wind but also the available thermal energy, i.e. the enthalpy.

The value of the power coefficient considering only the kinetic energy of the wind was compared with the respective value taking into account the enthalpy, the difference is only 4.56%, to determine the impact of this percentage it is recommended a new investigation in which the free wind data is compared with the experimental values in a wind tunnel.

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