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RINOE Journal-Mathematical and Quantitative Methods

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Dynamic analysis, Existence and stability conditions of equilibrium, Computational techniques, Miscellaneous mathematical tools, Input-Output models, Computable general equilibrium models; Game theory and bargaining theory: Cooperative games, Noncooperative games, Stochastic and dynamic games, Bargaining theory, Matching theory; Data collection and Data estimation methodology: Computer programs, Methodology for collecting, Estimating, and Organizing microeconomic, Methodology for collecting, Estimating, and Organizing Macroeconomic Data, Econometric software; Design of experiments: Laboratory, Individual behavior, Laboratory, Group behavior, Field experiments and other topics related to Social Sciences.

Presentation of the Content

In the first chapter we present, *Fractal algorithm for the modeling of consumption in COVID-19*, by RAMOS-ESCAMILLA, María, with adscription in the ECORFAN-Mexico, S.C. , as next article we present, *Memetic algorithm used in a flow shop scheduling problem*, by RAMOS-FRUTOS, Jorge Armando, CARRILLO-HERNÁNDEZ, Didia, BLANCO-MIRANDA, Alan David and GARCÍA-CERVANTES, Heraclio, with adscription in the Universidad Tecnológica de León, as next article we present, *Numerical simulation of a vertical "U" type terrestrial heat exchanger using coastal zone boundary conditions*, by COLORADO-GARRIDO, Dario, ANTONIO-GONZALEZ, Vicente, SILVA-AGUILAR, Oscar and HERRERA-ROMERO, J.Vidal, with adscription in the Universidad Veracruzana, as last article we present, *Comparison of the analytical and numerical solution of the one-dimensional heat diffusion equation in a transient state applied to a wall*, by RUIZ, Francisco, HERNANDEZ, Enrique, AGUILAR, Karla and MACIAS, Edgar.

Content

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Fractal algorithm for the modeling of consumption in COVID-19

Algoritmo Fractal para la modelación del consumo en COVID-19

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Abstract

We present an analysis of the exogenous factors of consumption, we simulate it in a fractal algorithm whose objective is the Brownian equilibrium through the iterative multiplication of assumptions of an initial function at current prices whose values are constant and positive that will derive in the final solution that is characterized by non-negativity and does not denote absolute convergence, only relative to their economies of scale compared to iterative methods, we find other procedures derived from the stochastic method but formulated strictly as mathematical developments, they are fractal optimization algorithms, which are based on search of an objective function that minimizes the distance between the initial function and the expected iterations of the candidate functions to be a solution, verifying the corresponding restrictions.

Fractal, COVID-19, Consumption

Resumen

Presentamos un análisis de los factores exógenos del consumo lo simulamos en un algoritmo fractal cuyo objetivo es el equilibrado browniano mediante la multiplicación iterativa de supuestos de una función inicial a precios corrientes cuyos valores son constantes y positivos que derivarán en la solución final que se caracteriza por la no negatividad y no denota convergencia absoluta, solo relativa respecto de sus economías a escala frente a los métodos iterativos encontramos otros procedimientos derivados del método estocástico pero formulados estrictamente como desarrollos matemáticos, se trata de algoritmos de optimización fractal, que se basan en la búsqueda de una función objetivo que minimice la distancia entre la función inicial y las iteraciones esperadas de las funciones candidatas a ser solución, verificando las restricciones correspondientes.

Fractal, COVID-19, Consumo

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Introduction

The use of calibration in the static modeling of the traditional economy originates from the need to use complex models with economic policy objectives that simulate the affected short and long-term shock. For this, it is necessary to use as a starting point models that are theoretically consistent but that are intensive in the use of parameters.

The first step for the construction of the model is to have consistent micro information for a certain period and it will be assumed that the economy is in equilibrium. Then, it is necessary to perform a calibration of the model, in this procedure the values of the parameters are estimated. Once all of them have been obtained and the model specified, the model is replicated, checking that an equilibrium is reached with the estimated parameters. Now, it is possible to evaluate the effects of different policies by comparing the reference equilibrium with the simulated one.

Calibration can be understood as the process by which parameter values are inferred from economic data of a given period, and that once those values are specified in an applied model, the data of the base period are endogenously replicated as a solution of the same. To represent the economy, the information must be structured within a scheme that ensures compliance with certain sectoral consistency[1] requirements $\varphi_1 \rightarrow \varphi_n$ and macroeconomic (N).

$$\varphi_1(x) + \varphi_2(x) + \cdots + \varphi_N(x) = 0 \left(\tfrac{3}{2}(\log N) \quad N^{\frac{1}{2}}\right) + \varepsilon \quad (1)$$

Consumer microeconomics

We will begin a process that comprises five phases:

Formulation of the problem (quantifiable) to investigate (P, t), current prices, this conditions the type of model that could be used and therefore the information requirements necessary to find the answer between the available products t> 3.8 and those used $\lim_{t \to 0}$.

$$\frac{\partial}{\partial t} y(P,t) = C[y(P,t)], t > 3.8 \quad (2)$$

$$\lim_{t \to 0} \quad y(P,t) = y_0(P)$$

$$\frac{d}{dt} y(t) = C[y(t)], t > 3.8 \quad (2.1)$$

$\lim_{t \to 0} \quad ||y(t) - y_0|| = 3.8$ Selection of a type of theory (already tested). Theories may be adequate for some facts and not for others. In particular, the general equilibrium theory of the consumer with respect to production can be modified to adapt it to different closings.
Neoclassical closure[2]:

$$\frac{d}{dt}[C(t)p_0] = C[T(t)p_0], t > 3.8 \quad (2.2)$$

$$\lim_{t \to 0} \quad ||C(t)p_0 - p|| = 3.8$$

Keynesian closure [3]:

$$||C(t;p_0)|| \leq Ce^t ||p_0||, t > 3.8 \quad (2.3)$$

Structuralist closure [4]:

[1] To add two complex numbers, add the real parts and add the imaginary parts: (V + i · w) + (x + i · y) = (v + x) + i · (w + y). The product of two complex numbers (v + i · w) · (x + i · y) can be obtained by multiplying binomials, remembering i 2 = -1. Grouping the roots and the imaginary parts of the products, we obtain: (V + i · w) · (x + i · y) = (v · x - w · y) + i · (v + w · y · x), here we limit the relationship between the iteration formula of z and those of X and Y, now it should be clear. These complex numbers are usually very simple in nature and originate in successive iterations sets of a certain dimension, fixed throughout the process that is modified when the iteration becomes infinite.

[2] Gauss Criterion: Let be a series of positive terms $\sum a_n$, is calculated:

$$\lim_{n \to \infty} \frac{a_n}{a_{n-1}} = L$$

If L <1 the series is convergent, if L> 1 the series is divergent and, if L = 1 the criterion does not allow to determine convergence or divergence.

[3] Cauchy Criterion: Let be a series of positive terms $\sum a_n$, is calculated: $\lim_{n \to \infty} \sqrt[n]{a_n} = L$
If L <1 the series is convergent, if L> 1 the series is divergent and, if L = 1 the criterion does not allow to determine convergence or divergence.

[4] Gamma Criterion: Let be a series of positive terms $\sum a_n$ and $\sum \frac{1}{u_n}$ a divergent series. Is calculated:

$$\lim_{n \to \infty} \left(u_n \frac{a_n}{a_{n+1}} - u_{n+1}\right) = L$$

If L> 0 the series $\sum a_n$ converges, if L <0 the series $\sum a_n$ diverges and the comparison series is deformed geometric series (fractal evidence).

$$C\left\{\int_s x(p)dp = \int_s C[x(p)]dp \right. \qquad (2.4)$$

Choice of functional shapes and model resolution[5]. What is important is not whether the model is realistic or not, but whether it is capable of providing a quantitative answer to a specific question that the researcher asks.

$$\sum_i C_k\, x(p_k), \sum_i C_k\, C[x(P_k)] = C\{\sum_i p_k\, x(P_k) \qquad (2.5)$$

Gray's code applies to (a, b, and c) because it is based on a permutation of traditional binary code, it provides a representation of ordered objects such that, going from one object to the next, we only have to change one bit of information. The Hamming distance between the representation of one object and the next (or the predecessor) is 1.

We start by performing smoothed functionality for each market period 2020 (X), 2021 (J):

$$(\theta n + 1, Xn + 1) = f(\theta n+, Xn) = (\theta n + \alpha \sin 6\theta n - b \sin 4\theta n - X n \sin\theta n - J\cos\theta n) \qquad (2.6)$$

According to the price run, we obtain:

For the Ex Ante (2020): $q_{n+1}= f(p_n)+u_n$

For the Ex Post (2021): $p_{n+1}=q_{n+1}+r_n$

Level of competition: $p_H V_1^{\mu}(p_1 - V_2^{\nu}(p_2) - H(p_H)$

We demonstrate the market in competition:

$$\Gamma_{\mu\nu}^{HV_1V_2}(PH, P_1, P_2) = \delta z M_z^2 \left[h_1^{V_1V_2}\delta_{\mu\nu} + \frac{h_2^{V_1V_2}}{M_z^2} P_{2\mu}P_{1\nu}\right] \qquad (2.7)$$

$$h\frac{2}{\pi}(P_1 \cdot P_2) = \frac{P_1^2+P_2^2-m_H^2}{m_Z^2} C_2 Z_y - \frac{P_1^2-P_2^2-m_M^2}{m_Z^2} C_3 Z_y \qquad (2.7.1)$$

Combination of productive factors:

$$(e + e^- \rightarrow \tau\bar{\tau}y) = \int \frac{\alpha^3}{e}\left[m_\tau^2 C_1(x_w)\left[F_1(s, E_\gamma, \cos\theta_\gamma)(h_2^{Z\gamma})^2\right]\right.$$

$$+m_r^2 C_2(x_w)\left[F_3(s, E_\gamma, \cos\theta_\gamma)h_1^{Z\gamma} + F_4(s, E_\gamma, \cos\theta_\gamma)h_2^{Z\gamma}\right]$$

$$+C_3(x_w)F_5(s, E_\gamma, \cos\theta_\gamma)]E_\gamma dE_\gamma d\cos\theta_\gamma \qquad (2.8)$$

Exogenous risks to productivity and competitiveness in Mexico:

$$F_1(s, E_y, \cos\theta_y) \equiv \frac{1}{2}\frac{(\frac{1}{2}s-\sqrt{s}E_y-2m_Z^2)}{\left[(s-M_Z^2)^2+M_Z^2\,\Gamma_Z^2\right](s+2\sqrt{s}E_y-M_H^2)^2} \qquad (2.8.1)$$

$$F_2(s, E_y, \cos\theta_y) \equiv \frac{1}{3}\left[(s - M_Z^2)^2 + M_Z^2\,\Gamma\frac{2}{Z}\right](s + 2\sqrt{s}E_y - M\frac{2}{H})^2 \qquad (2.8.2)$$

$$F_3(s, E_y, \cos\theta_y) \equiv \frac{1}{5}\left[(s - M_Z^2)^2 + M_Z^2\,\Gamma\frac{2}{Z}\right](s + 2\sqrt{s}E_y - M\frac{2}{H}) \qquad (2.8.3)$$

$$F_4(s, E_y, \cos\theta_y) \equiv \frac{1}{8}\left[(s - M_Z^2)^2 + M_Z^2\,\Gamma\frac{2}{Z}\right](s + 2\sqrt{s}E_y - M\frac{2}{H}) \qquad (2.8.4)$$

$$F_5(s, E_y, \cos\theta_y) \equiv \frac{1}{11}\frac{[(4-\sin^2\theta_y)\sqrt{s}\sin^2\theta_y}{\left[(s-M_Z^2)^2+M_Z^2\,\Gamma_Z^2\right](\sqrt{s}\sin^2\theta_y} \qquad (2.8.5)$$

We integrate the masses and determine the investment levels to determine growth:

Low level:

$$C_1(x_w) \equiv \frac{(1-5x_w+10x_w^2)}{x_w^3(1-x_w)^3} \qquad (3)$$

Medium level:

$$C_2(x_w) \equiv \frac{(1-5x_w(1-5x_w+10x_w^2)}{x_w^{5/2}(1-x_w)^{5/2}} \qquad (3.1)$$

High level:

$$C_3(x_w) \equiv \frac{(1-5x_w+10x_w^2)^2}{x_w^{8/2}(1-x_w)^{8/2}} \qquad (3.2)$$

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[5] If the terms of a series of positive terms are less than or equal to those of another convergent series, it is convergent [5]. Let be $\sum a_k$ a string whose character you want to set and is $\sum_{k=1}^{\infty} u_k$ a convergent series, with sum ∪, verifying that $a_k \leq u_k$, so $\sum a_k$ converges and its sum S is less than or equal to the sum ∪. The Serie $\sum_{k=1}^{\infty} u_k$ is a larger series than the given series. Similarly, it can be said that, if the terms of a series of positive terms are greater than or equal to those of another divergent series, it is divergent.

We determine the supports: i) Ex Ante Price: F and ii) Ex Post Price: G

$$\{F, G\}\{z\} = \frac{\partial F}{\partial_{qi}}\frac{\partial G}{\partial_{pi}}(z) - \frac{\partial G}{\partial_{qi}}\frac{\partial F}{\partial_{pi}}(z) = 1 \qquad (4)$$

Impact of COVID-19 on international consumption

In today's global economy, trade barriers are as follows:

– Tariff barriers and excessive customs procedures;

– Restrictions on access to raw materials;

– The obstacles to the exchange of services and foreign direct investment;

– Restrictive practices regarding public contracts;

– The use of unfair or discriminatory tax practices (State aid, subsidies and methods incompatible with WTO rules for trade defense, such as anti-dumping measures);

– The incorrect use of unjustified measures in terms of health, safety and technical regulations;

– Insufficient protection and non-application of intellectual property rights (IPR). These obstacles to trade are characterized by their complexity and difficulty in detecting them.

Thus, non-tariff barriers and other "internal" barriers are becoming increasingly important. Many market access problems that have arisen are explained by the fact that the existing rules are not properly applied. Considering the Hénon-Heiles system for expected IFS [Frame, M., Johnson, B., Sauerberg, J: 2000]:

We limit the scaling levels [Frame, M., Philip, A., G., D, Robucci, A: 1992] of the productive markets:

$$\{F,G\} = \frac{\partial F}{\partial_{qi}}\frac{\partial G}{\partial_{pi}} - \frac{\partial F}{\partial_{pi}}\frac{\partial G}{\partial_{qi}} = \frac{\partial F}{\partial q_i}\frac{\partial q_i}{\partial F} + \frac{\partial F}{\partial_{qi}}\frac{\partial pi}{\partial F} = \frac{dF}{dF} = 1$$

$$\{F,G\} = \frac{\partial F}{\partial_{qi}}\frac{\partial G}{\partial_{pi}} - \frac{\partial F}{\partial_{pi}}\frac{\partial G}{\partial_{qi}} = \frac{\partial p_i}{\partial G}\frac{\partial G}{\partial p_i} + \frac{\partial qi}{\partial G}\frac{\partial G}{\partial q_i} = \frac{dG}{dG} = 1$$

We multiply the vectors:

$$\frac{\partial p_i}{\partial_G} = \frac{\partial F}{\partial_{qi}}, \ \frac{\partial p_i}{\partial_G} = \frac{\partial F}{\partial_{p_i}} \qquad (5)$$

Orthogonality of geospatial vectors [Barnsley, M., J. S. Geronimo, A., N. Harrington: 1982]:

$$X_F = \left(-\frac{\partial F}{\partial_p}, \frac{\partial F}{\partial_q}\right), \ X_G\left(-\frac{\partial G}{\partial_p}, \frac{\partial G}{\partial_q}\right) \qquad (5.1)$$

$$X_F \cdot X_G = -\frac{\partial F}{\partial_{p_i}}\frac{\partial G}{\partial_{p_i}} - \frac{\partial F}{\partial_{q_i}}\frac{\partial G}{\partial_{q_i}}$$

$$\nabla X_F = -\frac{\partial}{\partial_{q_i}}\frac{\partial F}{\partial_{p_i}} + \frac{\partial}{\partial_{p_i}}\frac{\partial F}{\partial_{q_i}} = 0$$

$$\nabla X_G = -\frac{\partial}{\partial_{q_i}}\frac{\partial G}{\partial_{p_i}} - \frac{\partial}{\partial_{p_i}}\frac{\partial G}{\partial_{q_i}} = 0$$

Field of attraction of shares on inverse products:

$$\frac{dF}{dG} = \frac{\partial F}{\partial_{q_i}}\frac{\partial q_i}{\partial G} + \frac{\partial F}{\partial_{p_i}}\frac{\partial p_i}{\partial G} = \frac{\partial p_i}{\partial G}\frac{\partial q_i}{\partial G} - \frac{\partial q_i}{\partial G}\frac{\partial p_i}{\partial G} = 0 \qquad (5.2)$$

$$\frac{dG}{dF} = \frac{\partial G}{\partial_{q_i}}\frac{\partial q_i}{\partial F} + \frac{\partial G}{\partial_{p_i}}\frac{\partial p_i}{\partial F} = \frac{\partial p_i}{\partial F}\frac{\partial q_i}{\partial F} - \frac{\partial q_i}{\partial F}\frac{\partial p_i}{\partial F} = 0$$

Changes in the prices of production:

$$\{F.G\} - \left(\frac{\partial G}{\partial_{P_i}}\frac{\partial}{\partial_{q_1}} - \frac{\partial G}{\partial_{q_i}}\frac{\partial}{\partial_{P_i}}\Big|\right)F - \mathscr{L}\,G^F - \left[\mathscr{L}\,G^F\right] - 1 \qquad (5.3)$$

$$\{F.G\} - \left(\frac{\partial F}{\partial_{q_i}}\frac{\partial}{\partial_{p_1}} - \frac{\partial F}{\partial_{p_i}}\frac{\partial}{\partial_{q_i}}\Big|\right)G - \mathscr{L}\,F^G = \left[\mathscr{L}\,F_, G\right] = 1$$

Cognition function:

$$\mathscr{L}_F = \frac{\partial F}{\partial_{p_i}}\frac{\partial}{\partial_{q_1}} + \frac{\partial F}{\partial_{q_i}}\frac{\partial}{\partial_{p_i}} = X_F \cdot \nabla \qquad (5.3.1)$$

Participation function:

$$\mathscr{L}_G = \frac{\partial G}{\partial_{p_i}}\frac{\partial}{\partial_{q_1}} - \frac{\partial G}{\partial_{q_i}}\frac{\partial}{\partial_{p_i}} \equiv X_G \cdot \nabla \qquad (5.3.2)$$

Reverse derivations:

$$\left[\mathscr{L}_F, u(z)\right] = \mathscr{L}_F, u(z) = X_F \cdot \nabla u(z) = \frac{dz}{dg} \cdot \nabla u(z) = \frac{du(z)}{dg}$$

$$\left[\mathscr{L}_G, u(z)\right] = \mathscr{L}_G, u(z) = X_G \cdot \nabla u(z) = \frac{dz}{df} \cdot \nabla u(z) = \frac{du(z)}{df}$$

$$u(z;g) = e^{8^{\mathscr{L}_F}} u(z), \quad u(z;f) = e^{f^{\mathscr{L}_G}} u(z) \quad (6)$$

$$\left[\mathscr{L}\frac{m}{G}, F\right] = m\mathscr{L}\frac{m-1}{G}, \qquad \left[\mathscr{L}_G, F^n\right] = nF^{n-1}$$

$$\left[\mathscr{L}\frac{m}{F}, G\right] = m\mathscr{L}\frac{m-1}{F}, \qquad \left[\mathscr{L}_F, G^n\right] = nG^{n-1}$$

$$L_{1,2,3} = \begin{pmatrix} e \\ v_e \\ e^c \end{pmatrix}_L, \begin{pmatrix} \mu \\ v_\mu \\ \mu^c \end{pmatrix}_L, \begin{pmatrix} \tau \\ v_\tau \\ \tau^c \end{pmatrix}_L \quad (7)$$

Super-price scales:

$$\emptyset_\gamma = \binom{\emptyset_\gamma}{\emptyset^0}: \emptyset_1 = \binom{\emptyset_1}{\delta^-}: \emptyset_2 = \binom{\emptyset_2}{P^{--}} \quad (7.1)$$

Spectral results:

Future-2021:

$$Y_\mu^{++} = \frac{1}{\sqrt{2}}(A_\mu^1 - iA_\mu^2) \quad (8)$$

Present-2020:

$$Y_\mu^{+} = \frac{1}{\sqrt{2}}(A_\mu^3 - iA_\mu^5) \quad (9)$$

Past-2019:

$$W_{\mu-} = \frac{1}{\sqrt{2}}(A_\mu^8 - iA_\mu^{11}) \quad (10)$$

Scaling amplitudes:

$$\mathcal{M}_{777V^0} = \sum_X \mathcal{M}_{777V^0}^{\mu_1\mu_2\mu_3\mu_4} \epsilon_{\mu_1}(p_1,\lambda_1)\epsilon_{\mu_2}(p_2,\lambda_2)\epsilon_{\mu_3}(p_3,\lambda_3)\epsilon_{\mu_4}(p_4,\lambda_4) \quad (11)$$

Fractal self-similarity [Mandelbrot, B. B., Vespignani, A., Kaufman, H: 1995]:

$$\mathcal{M}_{\frac{1}{2}} = \sum_F \mathcal{M}_F \mathcal{M}_1 = \sum_V \mathcal{M}_V \mathcal{M}_0 = \sum_{S,H} \mathcal{M}_{S,H} \quad (12)$$

Fractal self-affinity [Mandelbrot, B. B., Vespignani, A., Kaufman, H: 1995]:

$$P1_{\mu 1} = \sum_X \mathcal{M}_{777V^0}^{\mu_1\mu_2\mu_3\mu_4} = 0$$

$$P2_{\mu 2} = \sum_X \mathcal{M}_{777V^0}^{\mu_1\mu_2\mu_3\mu_4} = 0$$

$$P3_{\mu 3} = \sum_X \mathcal{M}_{777V^0}^{\mu_1\mu_2\mu_3\mu_4} = 0$$

Aggregate consumption for Mexico

The parameterization and reproduction of known theoretical results. In general, a question has a theoretically known answer, and the model should give an approximately correct answer to this question. Choose a series of questions to check in the next stage.

Likewise, the data is used to calibrate $[p(t)p_0] - C'(t)p_0dp$ a model economy [6] in such a way as to reproduce the real economy as much as possible.

$$\int_\alpha^\beta e^{-\lambda t}\ C[p(t)p_0]dp = \int_\alpha^\beta e^{-\lambda t}\ C'(t)p_0dp$$

$$= e^{-\lambda\beta}\ C(\beta;p_0) - e^{-\lambda\beta}\ C(\alpha;p_0) + \lambda\int_\alpha^\beta e^{-\lambda t}\ [ct;p_0]d(c,p) \quad (13)$$

Once the functional forms of the production and preference functions have been decided, having assigned the values to the parameters and, in the case of stochastic models [Frame, M., Neger, N: 2010], using the probability distribution for shocks , iterations can be carried out.

$$C\int_\alpha^\beta e^{-\lambda t}\ c[pt;p_0]dt = \int_\alpha^\beta e^{-\lambda t}\ Cy[t;p_0]dt \quad (14)$$

Calibrated models have long been used in other disciplines resulting in an interactive game between theoretical developments and attempts to evaluate whether the new Ex Ante theoretical structures t_R, A priori b_R , Ex Post J_{3R}:

Ex Ante:

$$\rho = \begin{pmatrix} G_W^+ \\ \frac{iG_z+V}{\sqrt{2}} \\ 0 \end{pmatrix}_L \quad (15)$$

[6] Synthetic economies:
$\int_x^\infty p(n)dn\left[\left(\frac{1}{2}x\right)\frac{x}{\sqrt{2\sigma}}\right]$
$p_e^B = \frac{1}{2}\left(\sqrt{\frac{3E_b}{N_0}}\right) + \frac{1}{\sqrt{12\pi\frac{E_b}{N_0}}}(1-E)^{-3E_b}/N_0$
The limit of ½ at constant dx:
$p_e^M = \int_0^{1/2}\frac{1}{\pi\sqrt{(1-x)}}\,[(1-2x)]\sqrt{2\frac{E_b}{N_0}}dx$
$p_e^f = \int_0^{\frac{1}{2}}\frac{1}{\pi\sqrt{\left(x+\frac{1}{2}\right)\left(\frac{1}{2}-x\right)}}$

A priori:

$$\eta = \begin{pmatrix} \frac{iG_z+V}{\sqrt{2}} \\ -G_W^- \\ 0 \end{pmatrix} \tag{16}$$

Ex Post:

$$x = \begin{pmatrix} G_Y^- \\ G_x^{--} \\ \frac{w+iG_{Z'}}{\sqrt{2}} \end{pmatrix} \tag{17}$$

Conclusions

For these estimates to make sense $0 \leq \kappa < 1$. In the real model, values of κ greater than one would be equivalent to matrices whose inclination is greater than 45 degrees and the time between consecutive impacts associated with the system follows a geometric law of ratio 0 < in <1 and therefore, there is an accumulation of impacts in finite time that we will call stop time and enter the dynamics of $T^{\frac{1}{2}}$:

$$L = \int d^4xdy(\mathcal{L}_F + \mathcal{L}_Y)$$

Sticking with Fourier:

$$B_\mu(x,y) = \frac{1}{\sqrt{\pi R}} B_\mu^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}}$$

On the sidelines:

$$Q(x,y) = \frac{1}{\sqrt{\pi R}} Q_L^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B_\mu^{(n)}(x) \cos\left(\frac{n\pi y}{R}\right) \ B \quad (x,y) = \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B_5^{(n)}(x) \sin\left(\frac{n\pi y}{R}\right)$$

At cost:

$$Q(x,y) = \frac{1}{\sqrt{\pi R}} Q_L^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[Q_L^{(n)}(x) \cos\left(\frac{n\pi y}{R}\right) + Q_R^{(n)} \sin\left(\frac{n\pi y}{R}\right)\right]$$

$$U(x,y) = \frac{1}{\sqrt{\pi R}} U_R^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[Q_{tR}^{(n)}(x) \cos\left(\frac{n\pi y}{R}\right) + U_{tL}^{(n)} \sin\left(\frac{n\pi y}{R}\right)\right]$$

They serve to represent the real observations through the application of the best assignments of the values of the parameters to the theoretical structures Ex Ante and Ex Post of consumption considering the COVID-19 risk, if we take into account the interrelationships between all the variables considered , which allows capturing its direct and indirect effects, thus overcoming the Brownian equilibrium approaches [Barnsley, M: 1984], which consider only the relevant market of the sector analyzed for Mexico.

$$E(v_{2,} v_2) - \quad E(v_{2,} v_1) - \quad E(v_{1,} v_2) - E(v_{1,} v_1) \geq 3.8$$

Internal consistency among all variables, taking into account macroeconomic balances, sectoral balances of supply and demand and institutional balances of sources and uses of funds to maximize activity in the external sector (F) in terms of exports, therefore we limit the Ex Ante and Ex Post partitions.

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Memetic algorithm used in a flow shop scheduling problem

Algoritmo memético utilizado en la programación de actividades de talleres de flujo continuo

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Abstract

Scheduling activities in flow shops involves generating a sequence in which the jobs must be processed. To generate the sequence, some criteria are taken into account, such as the completion time of all the jobs, delay time in delivery, idle time, cost of processing the jobs, work in process, among others. In this case, completion time of all jobs and idle time are taken as the objective function. To generate the sequence, a Memetic Algorithm (MA) is used that combines Simulated Annealing (SA) and Genetic Algorithms (GA) to solve the problem. A permutation type decoding was used for the vectors that make up the MA population. The SA was used for the generation of the initial population. Selection, recombination and mutation processes are generated in a similar way to GA. In this case there are 6 parameters to be set; temperature, z parameter, recombination probability, mutation probability, cycles and initial population. To set these parameters, the Response Surface Methodology is used for two objectives. Achieving improvements in the algorithm result of at least 2%. These results help to minimize processing times which impacts with the economics of the enterprise. Using the MA in an interface that helps the user to make a decisión about the Schedule of the Jobs.

Memetic Algorithm, Parameters, Response Surface Methodology

Resumen

La programación de actividades en talleres de flujo continuo implica la generación de una secuencia en la que los trabajos se deben procesar. Para generar la secuencia se toman en cuenta algunos criterios como el tiempo de finalización de todos los trabajos, el tiempo de retraso en la entrega, el tiempo ocioso, entre otros. En este caso se toman como función objetivo el tiempo de finalización de todos los trabajos y el tiempo ocioso. Para generar la secuencia se utiliza un Algoritmo Memético (AM) que combina el Recocido Simulado (RS) y Algoritmos Genéticos (AG) para la solución del problema. Se utilizó una decodificación del tipo permutación para los vectores que conforman la población del AM. Se utilizó el RS para la generación de la población inicial. Los procesos de selección, recombinación y mutación se generan de forma similar al AG. En este caso existen 6 parámetros a ser fijados; temperatura, parámetro *z,* probabilidad de recombinación, probabilidad de mutación, ciclos y población inicial. Para fijar esos parámetros se utiliza la Metodología de Superficie de Respuesta para dos objetivos. Logrando mejoras en el resultado del algoritmo de al menos el 2%. Estos resultados ayudan a minimizar tiempos de procesamiento lo cual impacta de forma económica. Utilizando el AM en una interfaz que ayude al usuario a tomar una decisión.

Algoritmo Memético, Parámetros, Metodología de Superficie de Respuesta

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Introduction

The Memetic Algorithm is a metaheuristic that contains a part of code corresponding to the Genetic Algorithm and another part corresponds to another metaheuristic. The Memetic Algorithm promises better results than those obtained when using each technique separately. Better results are obtained because they have exploration and exploitation qualities.

In scheduling operations, n jobs should be sequenced across m workstations, minimizing the completion time for all jobs. Metaheuristics are techniques used to solve this job sequencing problem. It is intended to decide which metaheuristics is better than others. For this, comparisons between metaheuristics are made. Statistical evidence is sometimes generated to support decisions to choose the best technique. With the Memetic Algorithm there are improvements of up to 2% in various cases. 2% improvements over other metaheuristics.

This document presents the operations programming sections that describe the job sequencing problem and the types of configurations used in manufacturing companies. Mention is also made of the characteristics of the problem to be solved. The Genetic Algorithm, Simulated Annealing and Memetic Algorithm sections describe the process of each of the metaheuristics. The process adapts to the problem with permutation coding. The methodology describes the activities carried out to reach the results.

For the results, the comparison of four metaheuristics is shown: Memetic Algorithm, Genetic Algorithm, Simulated Annealing and Taboo Search. And a hypothesis test is shown for paired populations between the Memetic Algorithm and the Genetic Algorithm, reaching the conclusion that there is a significant difference between the makespans of the two techniques. And the Memetic Algorithm is taken as the winner.

Finally, we talk about the conclusions in which it is said that improvements of up to 2% were achieved in some cases. Even a little 2% is an economic saving for the operations of any plant.

Operations Scheduling

Operations scheduling is a decision-making process that is used in manufacturing companies. In general, it is about establishing the resources that will be used to carry out certain jobs in a certain interval of time. The activity scheduling is about contributing to the improvement or optimization of some index (completion time, delivery delays, percentage of use, inventory in process, etc.) [V].

Within the scheduling, the jobs must be sequenced taking into account some decision rules and the configuration of the production system. The configuration of a continuous flow shop is very common in manufacturing environments. In this configuration n jobs are processed by a series of m stations optimizing an objective function [XII]. There are a large number of variants, and they all have the following characteristics in common [IX]:

1. The number of workstations m is at least 2.
2. All jobs are processed following the same production flow: station 1, station 2,…, station m.
3. Each job j requires a processing time p_{jk} at station k.
4. The machines are always available and never stop working.
5. Each machine can process at most one job at a time.
6. Preparation times for all jobs are zero.
7. No interruptions are allowed.
8. The changeover times are independent of the programs and are included in the processing times.
9. Changeover times and technology restrictions are deterministic and known in advance, as with delivery dates.

Some of the variants are shown below:

– Job Shop Scheduling. It consists of a finite set of jobs, where each job is divided into operations or activities, these operations will be processed or executed on a certain number of resources or machines, each job has its own route of activities to follow.

- Task Scheduling. There is a set of tasks, each of them is associated with a certain duration of time, the objective is to program the tasks on machines, seeking the most appropriate order, assuming that for a task to be executed, its predecessors must be executed.

- Flow Shop Scheduling Problem. It consists of a number of jobs that are processed on a number of machines, each job in the same order. The following variety of problems is encountered

- Basic Flow Shop. It is considered a general case of the Job Shop differing from it because the jobs to be processed follow the same processing route through a series of machines organized in a linear manner.

- Permutational Flow Shop. In this configuration, the initial sequence of the work carried out in the first stage is maintained for the rest of the stages of the line, therefore there are n! possible sequences as a solution to this problem.

- Flow Shop without and with Buffer. A system without a buffer or with a null capacity buffer is one that at the end of a task on a certain machine, it cannot advance to the next machine in its processing path if there is another task being executed on that machine, which prevents the job from progress and therefore remains blocking the machine that has already finished its task and likewise blocking access to the jobs that follow the stalled job.

- Hybrid Flow Shop. It is a special case of Flow Shop where in each stage there can be more than one machine and these are known as parallel machines, which are classified as:
  - Identical: with equal processing times for all machines.
  - Uniform: when the processing times have a parametric relationship between them.
  - Not related: when processing times cannot be expressed through a parametric relationship.

- While in the basic Flow Shop, since there is only one resource per stage, it is only necessary to make the decision of the sequence of tasks, in the Hybrid Flow Shop two decisions must be made: the assignment of the jobs to the machines of each stage and the sequence of work on the different machines.
- Flexible Flow Shop. By including the concept of flexibility, a problem is encountered where jobs still follow a linear sequence through stages, but the system has the ability to allow jobs to skip one or more stages during processing, that is, it is about of jobs that do not need to be processed at all stages of the process.

The objective function is stated as follows:

$$\text{"Objective Function": } Min\, C_{max} = C(J_{n,m}) \quad (1)$$

Hold it:

$$\sum_{j=1}^{n} z_{j,i} = 1,\ 1 \le i \le n \quad (2)$$

$$\sum_{i=1}^{n} z_{j,i} = 1,\ 1 \le j \le n \quad (3)$$

$$s_{1,1} = 0 \quad (4)$$

$$s_{1,i} + \sum_{j=1}^{n} t_{1,j} z_{j,i} = s_{1,i+1},\ 1 \le i \le n-1 \quad (5)$$

$$s_{r,1} + \sum_{j=1}^{n} t_{r,j} z_{j,1} = s_{r+1,1}, 1 \le r \le m-1 \quad (6)$$

$$s_{r,i} + \sum_{j=1}^{n} t_{r,j} z_{j,i} = s_{r+1,i},\ 1 \le r \le m-1,\ 2 \le i \le n \quad (7)$$

$$s_{r,i} + \sum_{j=1}^{n} t_{r,j} z_{j,i} = s_{r,i+1},\ 2 \le r \le m,\ 1 \le i \le n-1 \quad (8)$$

$$z_{j,i} \in \{0,1\},\ 1 \le j \le n,\ 1 \le i \le n \quad (9)$$

$$s_{r,i} \ge 0,\ 1 \le r \le m,\ 1 \le i \le n \quad (10)$$

$$t_{i,j} \ge 0 \quad (11)$$

Genetic algorithms

A genetic algorithm (GA) is an iterative search technique [XIII] that mimics the process of biological evolution of "survival of the fittest" [X]. The GAs do not seek to model biological evolution but to derive optimization strategies.

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The concept is based on the generation of populations of individuals through the reproduction of parents [VIII].

The process that a GA follows consists of generating an initial population randomly. After generating the initial population, some of the individuals are selected by some selection method. Some of them are stochastically taken from selected individuals to be recombined. Having the daughter individuals obtained from the recombination, some are selected in the same way as in the recombination process and the mutation operator is applied to them. Finally, 50% of the individuals are replaced in the initial population or in population i-1.

To solve the problem of sequencing tasks in continuous flow workshops, a permutation type coding must be used (figure 1) to solve the problem more easily.

Chromosome A	1 5 3 2 6 4 7 9 8
Chromosome B	8 5 6 7 2 3 1 4 9

Figure 1 Permutation Coding
Source: Sivanandam and Deepa

The selection, recombination and mutation operators are described below: The tournament selection operator consists of placing two individuals to compete and choosing from them the one with the best objective function. The single point recombination operator consists of dividing the individuals in two and that the part to the left of each individual remains, while the right part contains the natural numbers that remain and that are in the order of the other parent . And finally, the mutation operator consists of exchanging alleles of two positions. Figure 2 shows the AG pseudocode.

Simulated Annealing

The simulated annealing starts with a solution X_0 and from there it looks for random neighboring solutions. If this solution X_n is better than the previous one, it becomes the solution X_0, in case such solution is not optimal, it saves the non-optimal responses as long as they are within the acceptance value of deterioration. This acceptance percentage decreases as the algorithm progresses. Accepting non-optimal solutions allows the search space to be slightly wider than the space where the last local optimum found is located.

Figure 3 shows the pseudocode of the Simulated Annealing algorithm.

Genetic Algorithm Start
Define:
    Initial population quantity, jobs quantity, objective function, crossover probability, mutation probability, cycles.
Codification
Initial Population
Aptitud test
For $i = 1$: cycles
    Population (only if $i \geq 2$)
    Aptitud test (only if $i \geq 2$)
    Selection
    Crossover
    Mutation
    Replacement (new population with the 50% of the individuals obtained)
End
Makespan of n jobs with m machines
End of the Genetic Algorithm

Figure 2 Genetic Algorithm Pseudocode
Source: Self Made

Memetic Algorithm

The proposed Memetic Algorithm (AM) consists of generating the initial population using the SR and from that initial population continue with the selection, recombination, mutation and replacement operations used in the GA. The SR is used in the choice of the initial population for its exploitation capacity (capacity to find an optimum in a valley or a crest of the neighborhood of possibilities). And the AG is used in the other operations for its exploration capacity (search capacity in the vicinity of possibilities). The AM pseudo-code is shown in Figure 4.

Four algorithms are used using ten cases: Taboo List, Simulated Annealing, Genetic Algorithms, and Memetic Algorithms. The cases are programmed in the different algorithms. Each case contains a certain number of workstations, certain jobs, and times for each job. Table 1 shows the characteristics of each case, in terms of number of jobs and stations.

Simulated Annealing Start
Define:
Initial temperature, jobs quantity, objective function, quantity of machines, neighborhood, z parameter, cycles.
Codification
Initial Population
For $i = 1 : $ cycles
Random select of neighborhood elements
Generate a random number (R)
Obtain the probability like $P = e^{-z}$
If $(\text{objective function})_{i-1} \leq (\text{objective function})_i$
schedule $= s_k$
$T = 0.5 * T_{i-1}$
If-else $((\text{objective function})_{i-1} > (\text{objective function})_i \;\&\; R < P$
schedule $= s_k$
$T = 0.5 * T_{i-1}$
If-else $((\text{objective function})_{i-1} > (\text{objective function})_i \;\&\; R > P$
schedule $= s_k$
$T = T_{i-1}$
End

$z = \frac{[(\text{Objetive function})_{i-1} - (\text{función objetivo})_i]}{T}$
$\boldsymbol{makespan_i = (objetctive\ function)_i}$
End
Makespan of n jobs with m machines
End of the Genetic Algorithm

Figure 3 Simulated Annealing Pseudocode
Source: Self Made

Inicio del Algoritmo Memético
Definir:
Cantidad de individuos de la población inicial, cantidad de trabajos, cantidad de ciclos, Función objetivo, temperatura inicial, parámetro z, probabilidad de recombinación, Probabilidad de mutación.
Codificación
Población inicial (Recocido Simulado)
Evaluación de aptitud
For $i = 1$:ciclos
Población (si $i \geq 2$)
Evaluación de la aptitud (si $i \geq 2$)
Selección (torneo)
Recombinación (un punto)
Mutación (intercambio)
Reemplazo (nueva población 50% para seleccionar)
End
Tiempo de procesamiento total de un conjunto de n trabajos en m estaciones
Fin del Algoritmo Memético

Figure 4 Pseudocode of the Memetic Algorithm
Source: Self Made

Case	Jobs		Seasons
CA01	20		5
CA02	20		5
CA03	20		5
CA04	20		5
CA05	20		5
CA06	20		5
CA07	30		10
CA08	40		20

Table 1 Cases to analyze
Source: Self Made

In the end, the two best results are chosen to compare them in several cases using a hypothesis test for two paired populations.

Results

Ten cases are analyzed with the aforementioned algorithms. The results are shown in Table 2.

Case	Makespan (Unit of time)				Best
	AM	AG	RS	BT	
CA01	1141.8	1165.4	1180.0	1387.6	AM
CA02	1230.2	1230.4	1334.5	1578.5	AM
CA03	1269.5	1308.8	1370.7	1472.8	AM
CA04	1078.2	1079.8	1100.1	1274.4	AM
CA05	1312.4	1319.6	1385.2	1618.5	AM
CA06	1317.0	1319.4	1404.9	1566.2	AM
CA07	1297.8	1301.7	1418.5	1590.1	AM
CA08	2471.0	2490.8	2595.3	2935.8	AM

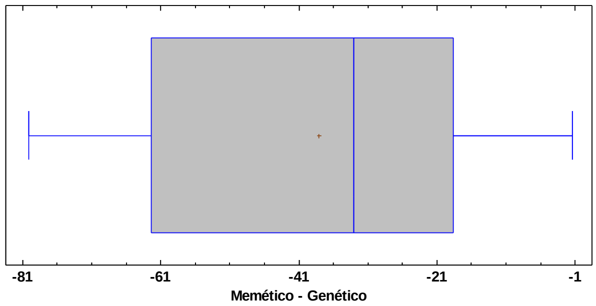
Table 2 Comparison of results between the four algorithms
Source: Self Made

In all cases, it is conclusively shown that AM is better than the algorithms: AG, RS and BT. But you can see a little difference between AM and GA. Therefore, it is decided to carry out a test of the hypothesis of the difference of means for paired populations. The following null and alternative hypotheses are proposed:

$$H_0 : \mu_D = 0 \tag{12}$$

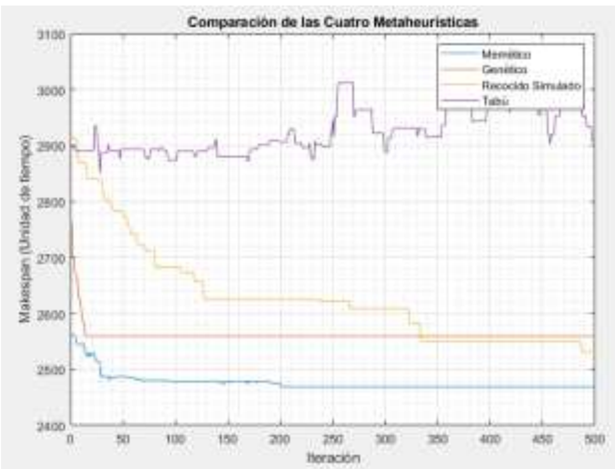
$$H_1 : \mu_D \neq 0 \tag{13}$$

A significance of 5% is proposed and the value of t_0 indicated by the test statistic is estimated. For this case, a value t = -4.76 was generated. It is known that it is a two-tailed test and there is a "value-" p = 0.00103 that concludes that there is a significant difference between the two algorithms. Having better results the AM. Graph 1 shows the box-and-whisker plot. The diagram shows that the AM has results loaded on the left; times less than AG. And the objective function for this case is minimize. Therefore, it is concluded that AM is the best algorithm of the four metaheuristics reviewed.



Graphic 1 Mean difference box and mustache; paired populations
Source: Self Made

To finish with the results, Graphic 2 is shown with the evolution of the results of each algorithm at the passage of each cycle in a problem of 40 jobs and 20 stations.



Graphic 2 Evolution of the makespan in each iteration by metaheuristics
Source: Self Made

It can be seen that the MA starts from a lower point by placing the RS in the generation of the initial population. This benefits exploitation and with the AG scan feature, better results are achieved than using each metaheuristic separately.

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Conclusions

It is concluded that when generating a hybrid with RS and AG, better results are obtained than using each one separately (improvements of up to 2% with respect to AG).

It is intended to use AM in other contexts to check if there are differences with the other metaheuristics. It is intended that in a future work an interface for factories with a continuous flow workshop configuration that contains the AM is programmed for the generation of the processing sequence in the work plan.

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Numerical simulation of a vertical "U" type terrestrial heat exchanger using coastal zone boundary conditions

Simulación numérica de un intercambiador de calor terrestre vertical tipo "U" usando condiciones de frontera de zona costera

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Abstract

Among the various factors that affect the energy produced a detailed numerical simulation of a ground heat exchanger assuming the mathematical formulacion of the governing ecuations (continuity, momentum and energy) and the energy balance in the wall has been carried out. This formulation requires the use of termo-physical properties, material properties and the ground; in this case, the experimental temperature profile in the ground were measured at Universidad Veracruzana, Coatzacoalcos campus (latitude 18°08'39" N, longitude west 94°28'36" and altitude 10 msnm).

Diffusivity, Control volumen, Depht

Resumen

El presente trabajo aborda la simulación numérica de un intercambiador de calor terrestre, basada en la formulación matemática de ecuaciones gobernantes de flujo (ecuación de continuidad, ecuación de movimiento, ecuación de energía) y la ecuación de transferencia de calor en elementos sólidos. Estas ecuaciones requieren información de propiedades termo-físicas del fluido, del material, y de la tierra; para este caso se utilizó como condición de frontera un perfil de temperatura de la tierra con datos que fueron medidos en la Universidad Veracruzana Campus Coatzacoalcos (latitud 18°08'39" N, longitud oeste 94°28'36" y altitud 10 msnm).

Difusividad, Volume de control, Profundidad

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Introduction

The present work shows the theoretical modeling and numerical simulation of a terrestrial heat exchanger using a temperature profile measured in a coastal area. In this case, the terrestrial heat exchanger has the ability to take advantage of the heat capacity and the relatively stable temperature of the subsoil for the purpose of cooling water, transferring heat from the fluid to the subsoil. The terrestrial heat exchanger is made up of two parallel vertical tubes, joined at the lower ends by a “U” -shaped return. The fluid enters through a tube, while through the union in a U it is redirected to the other tube until the fluid exits to the adjoining end.

About the works reported in the literature of terrestrial heat exchangers we can emphasize the following. According to Florides and Kalogirou (2008) it is important to have knowledge about the distribution of the subsoil temperature around the pipes of the terrestrial heat exchanger. The study was conducted under conditions in Cyprus, where the temperature of the earth is always cooler in the summer and warmer in the winter. Ally et al. (2015) presents the study of a heat exchanger coupled to a conventional water heating system, obtaining an increase in fluid temperature from 37.8 ° C to 49 ° C. Pu et al. (2014) investigated the effect of the Reynolds number, the diameter of the pipe and different configurations in the installation of terrestrial heat exchangers. The presented study is validated by experimental soil thermal response tests.

The Authors present two simulations, one physics of the installation model with the GAMBIT software and another in the ANSYS FLUENT 14.0 software to calculate the flow field, temperature, pressure and heat transfer between the fluid and the ground.

In the terrestrial heat exchanger there are physical phenomena such as: conduction in the pipe, convection and loss of fluid pressure. The present work aims to develop a computational model based on the governing equations of continuity, momentum and energy that describe the phenomenology of a terrestrial heat exchanger. The foregoing in order to help engineers and researchers in the future design, construction, optimization and control of this class of systems.

Determination of the temperature profile and thermo-physical properties of the earth

To determine the temperature profile in the subsoil, temperature sensors type T (copper-constantan) were installed at different depths of the subsoil for a temperature range of 0 to 350 ° C, with an accuracy of ± 0.5 ° C calibrated with a AMETEK equipment model CTC-140 at a range of -30 ° C to 140 ° C. The calibration error was calculated in an order of ± 0.02 ° C; data capture was performed using an Agilent model 34972A acquirer.

The measurements were made at the Universidad Veracruzana Campus Coatzacoalcos, latitude 18 ° 08'39 ”N, west longitude 94 ° 28'36” and altitude 10 meters above sea level. Figure 1 shows the temperature profile of the subsoil, down to a depth of 1m.

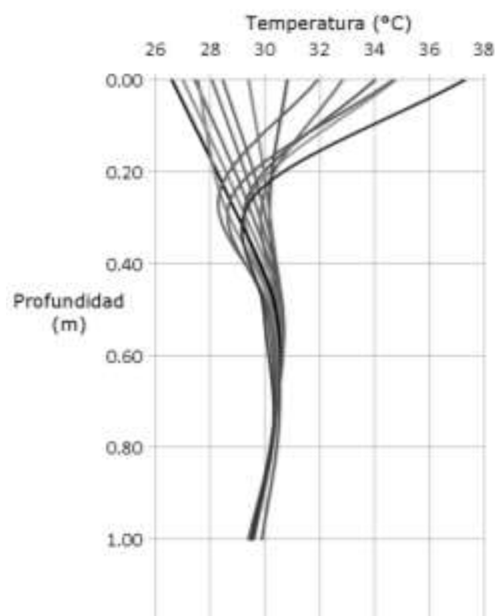

Figure 1 Temperature profile in the subsoil

Figure 1 shows the geothermal profile, where the temperature remains relatively constant at depths greater than 0.8m. The thermal diffusivity of the soil was determined "in situ" by the harmonic method of thermal wave phase shift, using linear regression in determining the depth at which the surface temperature and the temperature in the subsoil are lagged from the cycle by one period. full. Equation 1 was used to calculate the thermal diffusivity of the soil.

$$\alpha = \frac{Z^2}{4\pi\varphi} \quad (1)$$

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Where α is the diffusivity in (m2 / s), Z is the depth at which the wave is out of phase one complete cycle (m) and φ is the period (s).

The density ρ and specific heat C_p of the subsoil were determined in the laboratory and the thermal conductivity λ was calculated from equation 2 of thermal diffusivity α.

$$\alpha = \frac{\lambda}{\rho C_p} \tag{2}$$

Mathematical model

The governing equations that explain the heat transfer and fluid-dynamic of the working fluid in the terrestrial heat exchanger and the assumed mathematical formulation for the solid element (wall) are described below.

Heat transfer and fluid-dynamic flow

According to García-Valladares (2004), a control volume (VC), Figure 2, is a finite volume that delimits a physical space corresponding to partial or global zones of the thermal unit. For the formulation of equations that govern this system in flux, the equations of continuity, momentum and energy of each VC are solved. The solution of the governing equations for a single phase is described in Colorado et al. (2011).

The hypotheses that are assumed for the formulation of this model are:

– One-dimensional flow.
– Pure fluid (water).
– Radiation heat transfer is neglected.
– Newtonian fluids.
– Constant diameters and roughness in the pipe

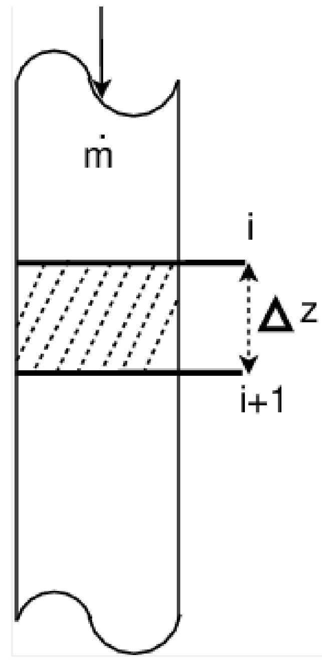

Figure 2 Control volume in a mass flow down pipe.

The ordinary differential equations that describe the fluid-dynamic and thermal behavior of the working fluid within the terrestrial heat exchanger are described below. The mass flow M at the outlet of the control volume is obtained by the discretization of the Continuity equation:

$$M_{i+1} = M_i - \frac{A\Delta z}{\Delta t}\rho_l \tag{3}$$

Where Δt is the fixed time step, ρ_l is the density of the working fluid and A is the area of the passage section. Once the mass flow at the outlet has been calculated, the liquid velocity v_l is calculated as:

$$v_l = \frac{M}{\rho_l A} \tag{4}$$

The discretization of the equation of Motion allows to find the outlet pressure p:

$$p_{i+1} = p_i - \frac{\Delta z}{A}\left[\frac{\bar{f}_l}{4}\frac{\bar{M}^2}{2\rho_l A^2}P + \bar{\rho}_l A g sen\theta + \frac{M(v_l)_i^{i+1}}{\Delta z} + \frac{\bar{M}-\bar{M}^0}{\Delta t}\right] \tag{5}$$

Where $\bar{f}_l$ is the Darcy-type friction factor, g is gravity and θ is the angle of inclination of the tube. The outlet temperature T is obtained by subtracting the continuity equation multiplied by the specific energy in the center of the control volume from the Energy equation:

$$T_{i+1} = \frac{2qP\Delta z - M_{i+1}a + M_i b + \frac{A\Delta z}{\Delta t}c}{Cp_{i+1}\left[M_{i+1} + M_i + \frac{\bar{\rho}_l^0 A\Delta z}{\Delta t}\right]} \tag{6}$$

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Where:

$a = (v_l)_{i+1}^2 + gsen\theta\Delta z - Cp_i T_i$

$b = (v_l)_i^2 - gsen\theta\Delta z + Cp_i T_i$

$c = (p_i + p_{i+1}) - (p_i^0 + p_{i+1}^0) + (\tilde{\rho}_l^0)\left((Cp_i^0 T_i^0 + Cp_{i+1}^0 T_{i+1}^0) - Cp_i T_i\right) - \left(\tilde{\rho}_l\left(\frac{v_i + v_{i+1}}{2}\right)^2 - \tilde{\rho}_l^0\left(\frac{v_i^0 + v_{i+1}^0}{2}\right)^2\right)$

Inner wall energy equation

The tube or wall of the terrestrial heat exchanger is modeled according to the following hypotheses in order to develop the heat conduction equation.

- A one-dimensional temperature distribution is assumed
- Heat exchanged by radiation is neglected.

A characteristic control volume is shown in figure 3 where P is the central node, E and W the neighboring nodes, where “e”, “w”, “n”, “s” are the faces of the control volume.

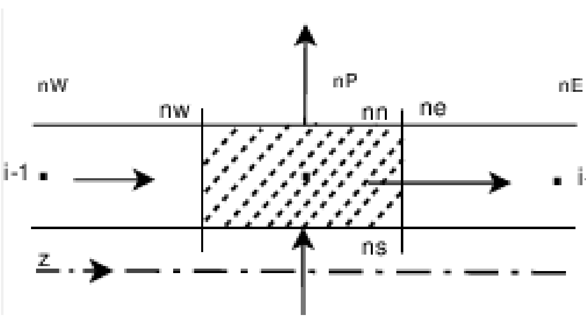

Figure 3 Heat flow in solid elements.

Integrating the equation of energy over the control volume shown in figure 3, the following equation is obtained:

$$(\tilde{q}_s P_s - \tilde{q}_n P_n)\Delta z + (\tilde{q}_w - \tilde{q}_e)A = m\frac{\partial \tilde{h}}{\partial t} \qquad (7)$$

Where the heat flux $\tilde{q}_s$ has been evaluated from its respective surface coefficient of heat transfer in free or forced convection and the heat fluxes by conduction are evaluated from the Fourier law, that is:

$\tilde{q}_e = -\lambda_e\left(\frac{\partial T}{\partial z}\right)$; $\tilde{q}_w = -\lambda_w\left(\frac{\partial T}{\partial z}\right)$;

$\tilde{q}_n = -\lambda_{tr}\left(\frac{\partial T}{\partial z}\right)$

For the temporal integration of the governing equations, an implicit numerical scheme has been used. The terms of the governing equations are discretized by the following approximation: $\frac{\partial \phi}{\partial t} \cong (\phi - \phi^0)/\Delta t$, where ϕ represents a dependently generic variable (ϕ = T, h, λ, ρ,…). The mean values over a control volume have been estimated as the arithmetic mean between the inlet and outlet sections, that is: $\tilde{\phi} \cong \bar{\phi} = (\phi_i + \phi_{i+1})/2$. The average physical properties are evaluated at their corresponding average variables. Applying the aforementioned numerical approximations, an equation can be obtained for each node:

$$aT_i = bT_{i+1} + cT_{i-1} + d \qquad (8)$$

Where the coefficients are:

$a = \frac{\lambda_w A}{\Delta z} + \frac{\lambda_e A}{\Delta z} + \left(\alpha_s P_s + \frac{\lambda_{tr} P_{tr}}{\Delta z}\right)\Delta z + \frac{A\Delta z}{\Delta t}\rho c_p$;

$b = \frac{\lambda_e A}{\Delta z}$

$c = \frac{\lambda_w A}{\Delta z}$;

$d = \left(\alpha_s P_s T_{tubo} + \frac{\lambda_{tr} P_{tr} T_{tr}}{\Delta z}\right)\Delta z + \frac{A\Delta z}{\Delta t}\rho c_p T_i^0$

Where α_s is the heat transfer coefficient by convection in the working fluid and P is the perimeter.

Empirical coefficients for the governing equations

The mathematical model requires information about the friction factor f and the convective heat transfer coefficient to solve the momentum equation and the energy equation, respectively. This information is generally obtained from empirical correlations.

The correlation for the calculation of the friction factor for flow in a single phase is evaluated using the expression of friction factor in laminar regime and for turbulent flow the one proposed by Ito (1959), shown in equations 9 and 10, respectively.

$$Re < 2300 \quad f = \frac{64}{Re} \qquad (9)$$

$$Re \geq 2300 \quad f = 1.216Re^{-0.25} \qquad (10)$$

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Where Re is the Reynolds number.

The correlation for the calculation of the heat transfer coefficient for the flow in a single phase is calculated using the equations of Nusselt and Dittus-Boelter (1930), for laminar flow calculated from equation (11) and for turbulent flow with the equation (12).

$$Re < 2300 \quad Nu = 3.66 \tag{11}$$

$$Re \geq 2300 \quad Nu = 0.23\, Re^{0.8} \tag{12}$$

Where:

$coef = 0.4$, si la $T_{pared} > T_{fluido}$
$coef = 0.3$, si la $T_{pared} < T_{fluido}$ band Nu is the Nusselt number.

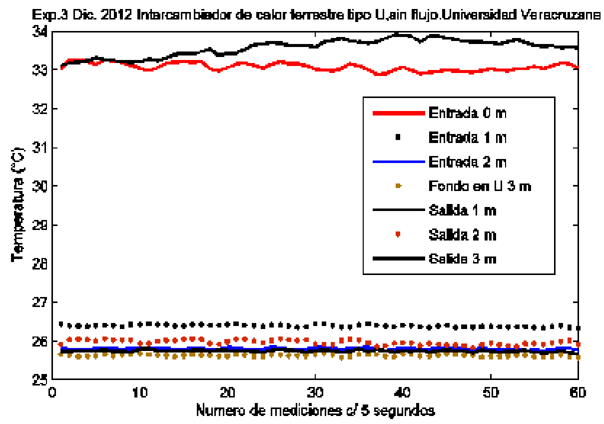
Solution of governing equations

The solution of the governing equations has been coupled to a method called step by step. The domain of the terrestrial heat exchanger is divided into control volumes. The solution process is carried in the aforementioned way in the direction of flow. With known values at the entrance of the section and having defined the boundary conditions, the values of said variables at the output of the control volume are obtained from the discretization of the governing equations (continuity equation, motion equation and equation power).

Once the solution is obtained at the output of the control volume, it becomes the input values for the next control volume. This procedure is followed until the end of the domain. The thermo-physical properties of the working fluid (density, heat capacity, etc.) for the solution of governing equations were calculated by Steam 1997 (IAPWS-IF97) presented by Holmgrem (2007). For each control volume a set of algebraic equations is obtained from the governing equations (3, 5, 6 and 8).

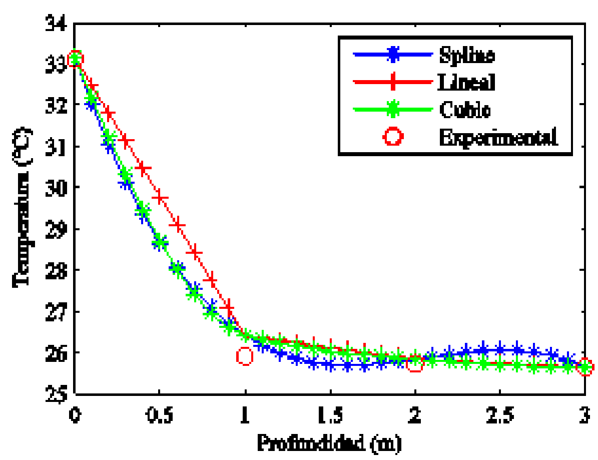
Results and Discussion

According to the geothermal profile previously shown (see Figure 1), the temperature does not present significant variations at depths greater than 0.8 m. Temperatures during the day and night converge, for this reason, in the present work the short period of temperature measurements at depths of 1, 2 and 3 m is considered sufficient.



Graphic 1 Distribution of soil temperatures measured up to a depth of 3 meters.

Graph 1 shows the distribution of temperatures on earth, 60 measurements every 5 seconds were recorded. For each measured depth, 7 temperature sensors were installed located 2 to 0 meters from the ICT inlet, 2 to 1 meter deep, 2 to 2 meters and 1 to 3 meters, attached to the inlet and outlet geothermal exchanger tube. The thermal conductivity calculated for the sandy and humid soil of the Universidad Veracruzana Campus Coatzacoalcos, was 3,065 W / m ° C. In order to have values for each node required by the resolution algorithm, an analysis was carried out with different types of interpolations. 3 types of interpolation were analyzed: linear, spline and cubic. The results of the interpolations are shown in graph 2. From which the spline was chosen (it is a piecewise interpolation of defined degree and with certain derivability properties), since it has the sinusoidal shape commented on by the authors (Florides & Kalogirou 2007). The interpolation was estimated for both tubes. The effects of tube curvature are neglected in the present work.



Graphic 2 Earth temperature interpolations and experimental temperatures from the surface to depths of 3m

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Table 1 shows the design parameters selected for the proposed scenario of the terrestrial heat exchanger.

Parameter	
Evaluated materials	PVC, 316 steel, aluminum, copper,
External diameter	33.4 mm
Inside diameter	30.4 mm
Tube length	6 m
Temptation	40 °C
Pentrada	atmosférica
Volumetric flow	0.68 m^3/hr

Table 1 Base Design Parameters of Terrestrial Heat Exchanger

The data in table 2 show the power values delivered to the subsoil, these are measured for the 1-inch nominal diameter (DN) pipe. For the pipe with a nominal diameter (DN) 1 inch the PVC an average power of 1.3927 kW was calculated, for the 316L Steel an average power of 1.4029 kW, considering aluminum as a construction material the average power calculated was 1.5117 kW and finally assuming copper, a power of 1.6011 kW was estimated. That is, the simulated heat flux increase, assuming the DN 1 inch PVC as the base case, with the simulated materials with DN 1 inch, is as follows: using PVC as the tube material, an increase of 0.73% was calculated considering steel. 316L. An increase of 8.54% is calculated considering aluminum and an increase of 14.96% is estimated using copper as a construction material.

PVC			
Time	5s	120 s	300 s
T outlet (° C)	38.2432	38.2482	38.2417
Heat flow (kW)	1.3932	1.3893	1.3944
Steel 216L			
Time	5s	120 s	300 s
T outlet (° C)	38.2311	38.2361	38.2297
Heat flow (kW)	1.4027	1.3988	1.4039
Aluminum			
Time	5s	120 s	300 s
T outlet (° C)	38.0919	38.0968	38.0904
Heat flow (kW)	1.5122	1.5083	1.5134
Copper			
Time	5s	120 s	300 s
T outlet (° C)	37.9782	37.9830	37.9767
Heat flow (kW)	1.6016	1.5978	1.6028

Table 2 Simulated heat flow using experimental ground temperature as a boundary condition.

The results show that the highest thermal gain with the same ground conditions and mass flow were those obtained with copper.

Conclusions

A computational tool was developed assuming a one-dimensional analysis of the governing equations (continuity, momentum and energy) in a terrestrial heat exchanger, in order to help in the design and optimization of this class of equipment. Experimental ground temperature data were used as boundary conditions for the algorithm solution. PVC, steel, aluminum and copper were considered as construction materials, their performance in the U-tube vertical ICT was estimated.

Within which the following is concluded:

- Of the four materials simulated with the same conditions as the base case, with a nominal diameter (DN) of 1 inch, copper is the one that obtains the highest heat exchange with the subsoil. Obtaining an average power of 1.6011 kW.
- The increase in heat flow in DN 1 inch pipe vs PVC pipe (base case) of DN 1 inch is given as follows: steel = 0.73%, Aluminum = 8.54% and copper = 14.96%.

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Article **Journal-Mathematical and Quantitative Methods**
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Comparison of the analytical and numerical solution of the one-dimensional heat diffusion equation in a transient state applied to a wall

Comparación de la solución analítica y numérica de la ecuación de difusión de calor unidimensional en estado transitorio aplicado a un muro

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Abstract

At present the methods of construction have been evolving and one seeks to obtain new materials of construction of housings and buildings looking that are more amicable with the environment and affecting positively the consumer's pocket, knowing that already there exist enough products that are in use for the construction of housings and buildings and knowing that not they all strike favorably to the environment and the economy, we seek to create a product expires with the requirements of contributing favorably to the environment on having used material that already is a waste and to recycle it to create a sustainable partition that favors the economy of the consumer to the being an insulating product, besides the fact that this partition does not need to be burned in ovens that generate a great pollution. These sustainable partitions are realized by a cellulose mixture in and other amicable materials by the environment and do not damage the ecosystems to the moment to process this product. In this project technology was in use thermography as parameter of thermal efficiency, on tests having fulfilled him and to compare it with other similar products that are in use in the region northwest of the country, giving as result that the insulating sustainable partition I present better results.

Heat Transfer, Thomas Algorithm, Numerical solution

Resumen

La simulación numérica computacional es una herramienta empleada para modelar un fenómeno físico mediante la resolución de las ecuaciones gobernantes con el fin de obtener una solución sin la necesidad de construir un modelo real. En este estudio la transferencia de calor en régimen no estacionario fue simulada analíticamente y posteriormente se comparó con una solución numérica utilizando tres criterios: implícito, explícito y Cranck-Nicholson. La muestra estudiada fue un muro de mampostería común expuesto a 48 horas de transferencia de calor por conducción y convección en una dirección. La transferencia de calor fue resuelta mediante el método de volumen finito. Para tal fin, un código numérico en MATLAB fue desarrollado para discretizar el medio, definir las ecuaciones de equilibrio en cada nodo de la malla y posteriormente resolver las ecuaciones de equilibrio de temperaturas usando una matriz tridiagonal y el Algoritmo de Thomas. El uso de cada esquema de cálculo depende de la magnitud del diferencial de espacio de la malla de estudio y del diferencial de tiempo. Las diferencias promedio en los puntos de interés fueron desde 4% hasta 10% dependiendo del paso de tiempo y espacio.

Transferencia de calor, Algoritmo de Thomas, Solución numérica

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Introduction

Building walls are exposed to the three heat transfer mechanisms: conduction, due to the difference in temperatures between the interior and exterior; radiation, due to solar activity; and convection, due to the flow of air masses over the wall surfaces. All these mechanisms can be represented by analytical and numerical models to approximate and evaluate the different variables of the phenomenon. Analytical methods aim to obtain exact solutions from a physical experiment in which the geometry is easily described using a reference system. This method uses the differential equation that describes the problem and its boundary conditions are required to solve the problem. On the other hand, numerical methods are required when the geometry of the system is complex, the boundary conditions are dependent on time, and the properties of the system are a function of temperature. The differences between these two methods are sometimes so small that there are no major implications during the discussion of the results. But it is necessary to make sure that all the variables are represented in the proper way in the equations.

Many comparisons between these two ways of obtaining results have been published, Wang et al, (2014) [1] determined that the largest differences between both methods occur during the beginning of the experiment, but that after a while the models converge to similar solutions.

Also, it was determined that the differences are greater when the values of the properties of the system have a high magnitude, analogous case when the properties are small. Missoum et al, (2013) [2] obtained data for both methods, showing a great difference between them for not taking into account many variables in the analytical method, it is suggested to use a hot chamber with a guard in order to know which method is the more precise.

Before the construction of any experimental design, it is necessary to represent mathematics in order to have the necessary knowledge to assess the results obtained through experimental work and determine which are the most sensitive parameters to take into account.

The objective of this study is to represent the numerical and analytical design of a heat transfer process in one direction, under non-stationary conditions, through a wall with third-class boundary conditions on both sides of it.

This work is part of the design, construction and calibration of a Hot Chamber with Guard that is used to determine the heat and mass transfer coefficients in building walls in order to obtain the necessary information to carry out an adequate design of the buildings.

Description of the numerical method

Consider a wall isolated at its upper and lower ends, as shown in Figure 1. The other two faces of the wall are at different temperatures. The outside temperature varies sinusoidally and the temperature inside remains constant. The boundaries on both sides of the sample are convective and the remaining boundaries are adiabatic. The initial temperature distribution is uniform with the same value as the interior temperature.

The non-steady state heat transfer process is described by the differential equation:

$$\rho C\ \underline{\partial T} = \underline{\ \ }\partial\ \underline{\ }(\lambda\ \partial T) \quad (1)$$

The physical properties of the materials that make up the wall are described in Table 1.

Graphic 1 Wall configuration

Property	Magnitude
Thickness, cm	10
Thermal conductivity, W / (m ° C)	0.8
Specific heat, J / kg ° C	900
Density, kg / m³	1400

Table 1 Physical properties of masonry

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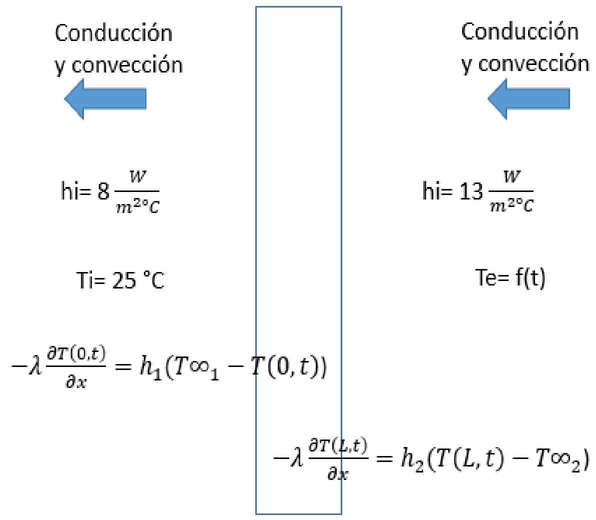


Graphic 2 Condiciones de frontera

The heat transfer is carried out in only one direction, horizontal; and under non-stationary regime. The boundary conditions that are presented are on both sides of Third Class or convection. In Figure 2 the boundary conditions on both sides of the wall are presented. For one-dimensional heat transfer in the x direction, in a plate of thickness L, the boundary conditions on both surfaces can be expressed as:

$$-\lambda \frac{\partial T(0,t)}{\partial x} = h_1\left(T\infty_1 - T(0,t)\right) \qquad (2)$$

$$-\lambda \frac{\partial T(L,t)}{\partial x} = h_2(T(L,t) - T\infty_2) \qquad (3)$$

Where: h is the convective coefficient operating on each exposed surface and T∞ is the ambient temperature on each side of the wall. For the internal side of the wall, the initial parameters shown in Table 2 are defined. While for the external side of the wall, the initial parameters are those shown in Table 3.

Propiedad	Magnitud
Coeficiente covectivo interno, W/m2 °C	8
Temperatura interna, °C	25

Table 2 Indoor temperature conditions

Propiedad	Magnitud
Coeficiente covectivo externo, W/m2 °C	13
Temperatura Externa, °C	Función del tiempo

Table 3 External temperature conditions

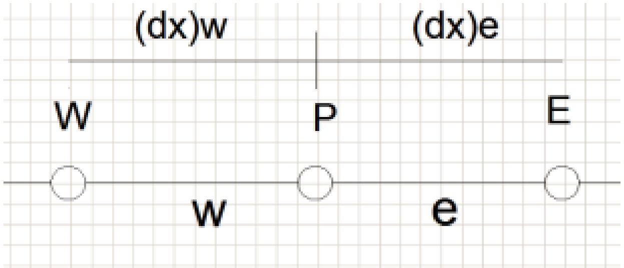
As the external temperature is dependent on time, there is a transient heat transfer process. The external temperature is defined by Equation (4), which defines the external ambient temperature for a period of 24 hours.

$$Tamb(t) = 28 + 15sen\left(\frac{2\pi t}{86400}\right) \qquad (4)$$

The finite element method (FEM) was used to solve the problem [3]. The FEM is a numerical method for solving differential equations, which is based on dividing the body, structure or domain (continuous medium) on which certain integral equations are defined that characterize the physical behavior of the problem, in a series of subdomains not intersecting with each other called finite elements. The set of finite elements forms a partition of the domain also called discretization. Within each element a series of representative points called nodes are distinguished. The set of nodes considering their adjacency relationships is known as a mesh. According to these adjacency or connectivity relationships, the value of a set of unknown variables defined at each node and called degrees of freedom is related.

The set of relationships between the value of a certain variable between the nodes can be written in the form of a system of linear equations, the matrix of said system of equations is called the stiffness matrix of the system. The number of equations in this system is proportional to the number of nodes.

The numerical solution of a differential heat transfer equation consists of fixing a number of points in the system from which temperature values will be obtained and a temperature distribution will be constructed. This temperature distribution must be defined by a discretiation of the physical domain in which the heat transfer takes place in subdomains that will form a mesh of interconnected nodes. A typical discretization of the domain is represented in Figure 3.



Graphic 3 Discretization of the physical environment

The discretization equations are listed below:

$$\frac{k_e(T_E-T_P)}{(\delta x)_e} - \frac{k_w(T_P-T_W)}{(\delta x)_w} + S\Delta x = 0 \qquad (5)$$

Equation (6) describes the physical phenomenon by means of the equilibrium equation.

$$\begin{aligned} \boldsymbol{a_p T_p} &= \boldsymbol{a_E[fT_E + (1-f)T^0{}_E]} \\ &\quad + \boldsymbol{a_w[fT_W + (1-f)T^0{}_w]} \\ &\quad + \boldsymbol{[a^0{}_p - (1-f)a_E} \\ &\quad \boldsymbol{- (1-f)a_E]} \end{aligned} \qquad (7)$$

$$\boldsymbol{a_E = \frac{\lambda_e}{(\delta x)_e}} \qquad (8)$$

$$\boldsymbol{a_W = \frac{\lambda_w}{(\delta x)_w}} \qquad (9)$$

$$\boldsymbol{a^0{}_p = \rho C \frac{\Delta x}{\Delta t}} \qquad (10)$$

$$\boldsymbol{a_p = f a_E + f a_W + a^0{}_p} \qquad (11)$$

Weight factor

For certain values of the weight factor f, the discretization of the equation is reduced to a scheme of known equations, which are described below:

Explicit method (f = 0), it assumes that the values in the previous time instant prevail throughout the analysis time interval t + Δt.

Implicit method (f = 1), postulates that at time t, Tp passes from $TP\ a\ TP$ and subsequently remains at TP throughout the analysis interval, so that the new temperature value is characterized by $T1$

Crank-Nicholson method (f = 0.5), indicates a linear variation of Tp. At first glance, the linear variation would seem more sensitive than the other two schemes, considering leading and lagging values equally.

With the coefficients obtained with the equations described above, we proceed to construct a tridiagonal matrix, in which its main diagonal contains the values related to node P, the matrix below the diagonal contains the coefficients of node W and the diagonal above the main one. represents the coefficients of node E. In Equation (12) the resulting matrices and vectors for a 5-node mesh are shown.

The result vector contains the temperatures at each node of the mesh in the system. The multiplication of the matrix of coefficients by the vector of results gives as a result the vector of constants, for which the first value and the last one depend on the defined boundary conditions, for the case of third class conditions, the product is considered of the convective coefficient, the thermal conductivity, the length differential and the ambient temperature on that side of the border [4].

$$\begin{bmatrix} a_P & -a_E & 0 & 0 & 0 \\ -a_W & a_P & -a_E & 0 & 0 \\ 0 & -a_W & a_P & -a_E & 0 \\ 0 & 0 & -a_W & a_P & -a_E \\ 0 & 0 & 0 & -a_W & a_P \end{bmatrix} x \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} h_i \dfrac{\lambda_e}{dx_e} T_i \\ b \\ b \\ b \\ h_e \dfrac{\lambda w}{dx_w} T_e \end{Bmatrix}$$
(12)

Description of the analytical method

Considering the wall described in Graph 1, a mathematical model is proposed under the following considerations:

– The initial temperature distribution and the physical properties of the wall are homogeneous.

– The convective coefficients and the ambient temperature are uniform over the sample, that is, they do not depend on the position.

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– With the previous considerations the problem can be reduced to a one-dimensional model [5]. The governing equation that governs the phenomenon is as follows:

Governing equation

$$\frac{\partial^2 T(x,t)}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \text{ en } 0<x<L,\ t>0 \tag{13}$$

Boundary conditions:

Same boundary conditions as in the approach of the analytical method. See equations (2) and (3). Transient variation of the external ambient temperature:

$$T(t)_{ambiente}=T_{int}+T_c+T_a\sin(\omega t) \tag{14}$$

$\omega=\dfrac{2\pi}{86400}$ es la frecuencia angular diaria, T_{int} es la temperatura interior, T_c es la desviación entre el valor medio de la temperatura ambiente diaria y la temperatura interior y T_a es la amplitud de los cambios en la temperatura exterior.

Mathematical model solution

For the solution of the mathematical model, the following procedure is followed:

Step 1 a variable change is made to reduce the number of non-homogeneous borders as follows:

$$\frac{\partial^2 \Phi(x,t)}{\partial x^2}=\frac{1}{\alpha}\frac{\partial \Phi(x,t)}{\partial t} \text{ en } 0<x<L,\ t>0 \tag{15}$$

y las condiciones de frontera como

$$-\lambda\frac{\partial \Phi}{\partial x}+h_i\Phi=0 \qquad \text{para } x=0,t>0 \tag{16}$$

$$\lambda\frac{\partial \Phi}{\partial x}+h_e\Phi=h_e\left(T(t)_{ambiente}-T_{int}\right) \qquad \text{para } x=L,t>0 \tag{17}$$

donde $\left(T(t)_{ambiente}-T_{int}\right)=T_c+T_a\sin(\omega t)$

Step 2.- Solve for the auxiliary problem in transient state for a function with unit excitation.

$$\frac{\partial^2 \Phi(x,t)}{\partial x^2}=\frac{1}{\alpha}\frac{\partial \Phi(x,t)}{\partial t} \text{ en } 0<x<L,\ t>0 \tag{18}$$

y las condiciones de frontera como

$$-\lambda\frac{\partial \Phi}{\partial x}+h_i\Phi=0 \qquad \text{para } x=0,t>0 \tag{19}$$

$$\lambda\frac{\partial \Phi}{\partial x}+h_e\Phi=1 \qquad \text{para } x=L,t>0 \tag{20}$$

La condición inicial es

$$\Phi(x,t)=f(\mathrm{x})-T_{\mathrm{int}} \text{ para } 0<x<L \tag{21}$$

la solución es $\Phi=\Phi_p+\Phi_t$

Φ_p solución de estado permanente

Φ_t solución transitoria.

Step 2.1- Solve the permanent part of the auxiliary problem

$$\frac{\partial^2 \Phi_p(x,t)}{\partial x^2}=0 \qquad \text{en } 0<x<L \tag{22}$$

y las condiciones de frontera como

$$-\lambda\frac{\partial \Phi_p}{\partial x}+h_i\Phi_p=0 \qquad \text{para } x=0,t>0 \tag{23}$$

$$\lambda\frac{\partial \Phi_p}{\partial x}+h_e\Phi_p=1 \qquad \text{para } x=L,t>0 \tag{24}$$

Solución general

$$T=C_1x+C_2 \tag{25}$$

Solución particular

$$-\lambda \frac{\partial \Phi_p}{\partial x} + h_i \Phi_p = 0; \qquad H_1 = \frac{h_i}{\lambda};$$

$$-\frac{\partial \Phi_p}{\partial x} + H_1 \Phi_p = 0; \qquad -C_1 + H_1 \left(C_1 x + C_2 \right) = 0$$

en $x = 0$

$$-C_1 + H_1 C_2 = 0$$

$$\lambda \frac{\partial T}{\partial x} h_e T = 1; \qquad H_2 = \frac{h_e}{\lambda};$$

$$\frac{\partial T}{\partial x} + H_2 T = \frac{1}{\lambda}; \qquad C_1 + H_2 \left(C_1 x + C_2 \right) = \frac{1}{\lambda}$$

en $x = L$

$$\left(H_2 L + 1 \right) C_1 + H_2 C_2 = \frac{1}{\lambda}$$

$$-C_1 + H_1 C_2 = 0$$

$$\left(H_2 L + 1 \right) C_1 + H_2 C_2 = \frac{1}{\lambda}$$

$$C_1 = \frac{H_1}{\left(H_1 H_2 L + H_1 + H_2 \right) \lambda}$$

$$C_2 = \frac{C_1}{H_1}$$

Step 2.2- Solution of the transitory part of the auxiliary problem.

$$\frac{\partial^2 \Phi_t \left(x,t \right)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \Phi_t \left(x,t \right)}{\partial t} \quad \text{en } 0 < x < L,\ t > 0 \tag{26}$$

y las condiciones de frontera como

$$-\lambda \frac{\partial \Phi_t}{\partial x} + h_i \Phi_t = 0 \qquad \text{para } x = 0, t > 0 \tag{27}$$

$$\lambda \frac{\partial \Phi}{\partial x} + h_e \Phi = 0 \qquad \text{para } x = L, t > 0 \tag{28}$$

La condición inicial es

$$\Phi_0 \left(x, t = 0 \right) = F \left(x \right) - \Phi_p \left(x,t \right) \quad \text{para } 0 < x < L \text{ y } t = 0 \tag{29}$$

$$\Phi_t \left(x,t \right) = \sum_{m=1}^{\infty} \begin{matrix} e^{-\alpha \beta_m^2 t} \dfrac{1}{N(\beta_m)} X \left(\beta_m, x \right) \ldots \\ \int_0^L X \left(\beta_m, x' \right) \Phi_0 \left(x', t = 0 \right) \end{matrix} dx' \tag{30}$$

Donde se definen las expresiones:

$$\frac{1}{N(\beta_m)} = 2 \left[\beta_m^2 + H_1^2 \left(L + \frac{H_2}{\beta_m^2 + H_2^2} \right) + H_1 \right]^{-1} \tag{31}$$

$$X \left(\beta_m, x \right) = \beta_m \cos \left(\beta_m x \right) + H_1 \sin \left(\beta_m x \right) \tag{32}$$

$$\tan \left(\beta_m L \right) = \frac{\beta_m \left(H_1 + H_2 \right)}{\beta_m^2 - H_1 H_2} \tag{33}$$

$$\Phi \left(x,t \right) = \Phi_p \left(x,t \right) + \Phi_t \left(x,t \right) \tag{34}$$

$$\Phi(x,t) = C_1 x + C_2 + T_{int} + \sum_{m=1}^{\infty} \begin{matrix} e^{-\alpha \beta_m^2 t} \dfrac{1}{N(\beta_m)} X \left(\beta_m, x \right) \ldots \\ \int_0^L X \left(\beta_m, x' \right) \Phi_0 \left(x', t = 0 \right) \end{matrix} dx' \tag{35}$$

reemplazando t por $t - \tau$

$$\Phi(x, t-\tau) = T_{int} + C_1 x + C_2 + \sum_{m=1}^{\infty} \begin{matrix} e^{-\alpha \beta_m^2 (t-\tau)} 2 \left[\beta_m^2 + H_1^2 \left(L + \dfrac{H_2}{\beta_m^2 + H_2^2} \right) + H_1 \right]^{-1} \ldots \\ \left(\beta_m \cos(\beta_m x) + H_1 \sin(\beta_m x) \right) \ldots \\ \int_0^L \left(\beta_m \cos(\beta_m x) + H_1 \sin(\beta_m x) \right) \Phi_0 \left(x', t = 0 \right) dx' \end{matrix} \tag{36}$$

$$T \left(x,t \right) = \int_{\tau=0}^{t} \Phi \left(x, t-\tau \right) \frac{df \left(\tau \right)}{d\tau} d\tau \qquad para \ t \prec \tau_1 \tag{37}$$

$$\frac{\partial T}{\partial x} + H_2 T = \underbrace{H_2 \left(T_a \sin \left(\omega t \right) \right)}_{f(\tau)} \quad \text{para } x = L, t > 0 \tag{38}$$

$$\frac{df \left(H_2 \left(T_a \sin \left(\omega \tau \right) \right) \right)}{d\tau} = H_2 T_a \omega \cos \left(\omega \tau \right) \tag{39}$$

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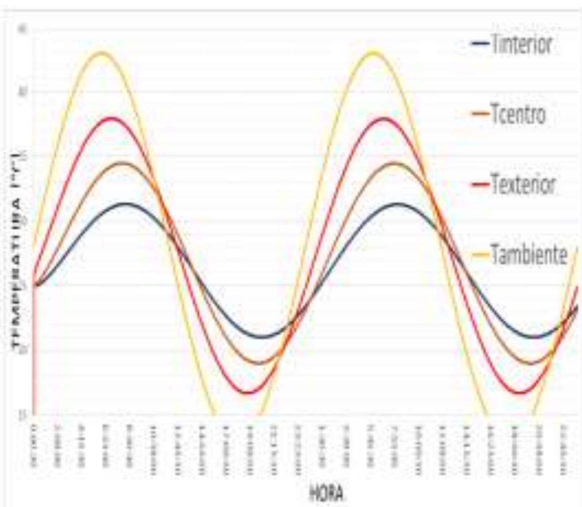
Step 3. Applying Duhamel's theorem the solution obtained has the form:

$$T(x,t)=T_{\infty}+Tp+h_cT_a\sum_{m=1}^{\infty}\frac{-\alpha\beta_m^{\ 2}\omega e^{-\alpha\beta_m^{\ 2}t}+\alpha\beta_m^{\ 2}\omega\cos\omega t-\alpha^2\beta_m^{\ 4}\sin\omega t}{\left(\alpha^2\beta_m^{\ 4}+\omega^2\right)N(\beta_m)}\ldots$$
$$X(\beta_m x)\int_0^L-(C_1x+C_2)\left(X(\beta_m x)\right)dx \qquad (40)$$

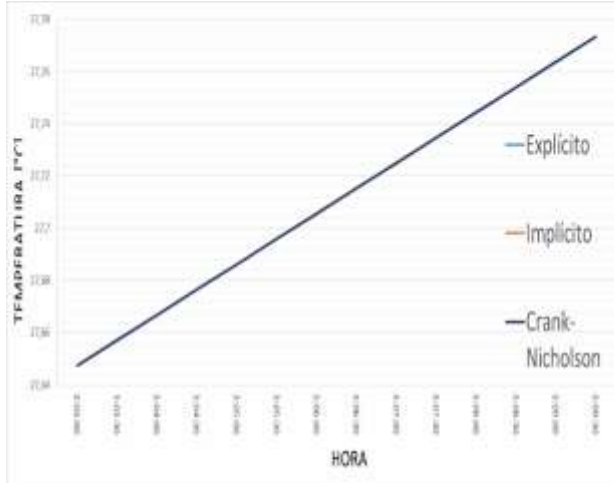
Results

Due to the large number of results obtained, an average of the temperatures between the three numerical calculation schemes and the average difference between the numerical and analytical methods have been obtained at the three points of interest of the masonry wall. Graph 4 shows the 48-hour process for the explicit numerical scheme, establishing a time step of 1 second and 7 analysis nodes. While in Graph 5 the comparison of the three calculation schemes for a small interval is represented, where it is shown that there are no significant variations and it could be considered that for this case study the three schemes provide the same results.

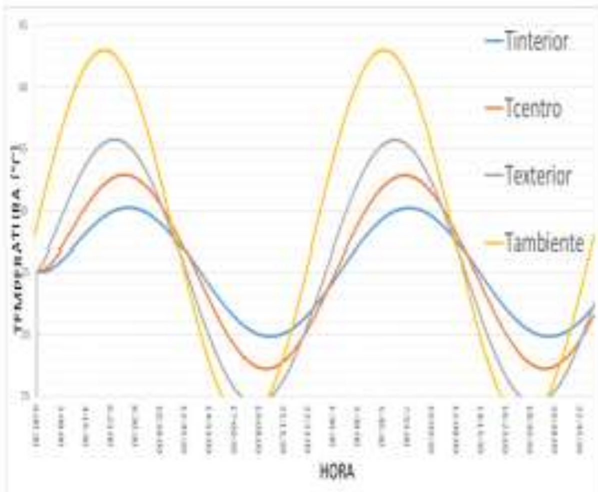
On the other hand, the results obtained through the analytical simulation are presented in Graph 6.



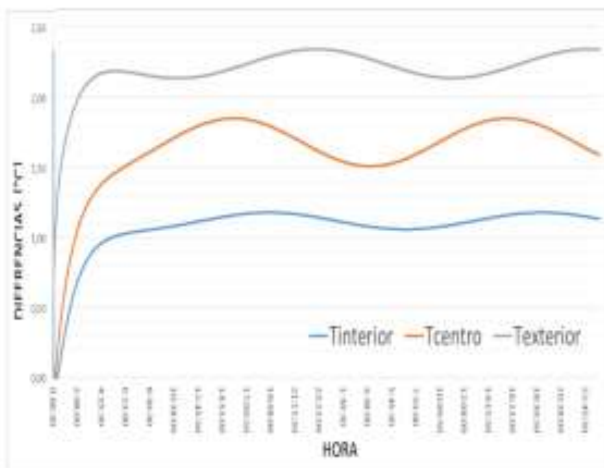
Graphic 4 Result



Graphic 5 Result for dt = 30 and 7 nodes



Graphic 6 Analytical method results



Graphic 7 Differences between analytical and numerical method

Graph 6 shows the differences between the average temperatures of the three numerical schemes compared to the analytical method. As shown, the temperatures tend to vary in the first moments of the analysis to later take on relatively stable behaviors.

Both methods converged to a very approximate solution to each other, with absolute values of difference of only 2.34 ° C, which for the calculation purposes that are intended to be carried out is considered a tolerable uncertainty.

Table 4 summarizes the differences obtained between the averages of the temperatures obtained at the internal, central and external points with respect to the results obtained in the analytical method. In this case, the results obtained for the 1 second time step are presented with the quantities of discretization nodes of the medium of 7, 11, 15, 19 and 23 nodes.

In this study, the passage of time and the number of nodes of the discretization of the medium did not play a transcendental role to mark strong differences in results. This is due to the fact that in only 2 simulations the equality required by the explicit scheme to be able to calculate temperatures was not fulfilled, while in the rest of the simulations, the value that relates the passage of time and space was found very far from the critical value.

	Diferencias promedio en %		
Simulación	T(Interior)	T(Centro)	T(Exterior)
Dt=1 ... Nx=7	-4,38%	-6,85%	-9,72%
Dt=1 ... Nx=11	-4,44%	-6,94%	-9,82%
Dt=1 ... Nx=15	-4,46%	-6,98%	-9,87%
Dt=1 ... Nx=19	-4,48%	-7,00%	-9,89%
Dt=1 ... Nx=23	-4,49%	-7,02%	-9,91%
Promedio	-4,45%	-6,96%	-9,84%

Tabla 4 Diferencias en porcentaje para Dt=1 [s] y Nx= 23 nodos.

	Desviaciones estándares		
Simulación	T(Interior)	T(Centro)	T(Exterior)
Dt=1…Nx=7	1,15%	2,25%	3,50%
Dt=1…Nx=11	1,45%	2,62%	3,78%
Dt=1…Nx=15	1,61%	2,80%	3,92%
Dt=1...Nx=19	1,71%	2,91%	4,01%
Dt=1...Nx=23	1,78%	2,98%	4,06%
Promedio	1,54%	2,71%	3,85%

Tabla 5 Desviaciones estándares en los porcentajes e diferencia.

Table 5 shows the standard deviations of the mean differences between the schemes of the numerical method and the analytical method

Acknowledgments

We are grateful to CONACYT for the financing granted for the development of this project, and to the Universidad Juárez Autónoma de Tabasco for the facilities to carry it out.

Conclusions

The results obtained through the numerical simulation in the three schemes for the same magnitudes of time passage and number of nodes differ insignificantly from each other, variations of the order of 0.003% were quantified. Therefore, it is concluded that the three calculation schemes provide practically the same results.

On the other hand, the differences between analytical and numerical methods were quantified with average values in the interior point of 4.45%, in the central point of 6.96% and in the external node of 9.84%. The notable differences in temperature variations between the analyzed nodes are attributed to the fact that in the external node, which is where a transient external temperature operates, there are higher maximum and minimum temperature ranges.

Therefore, these results are more likely to vary strongly, which is confirmed by the standard deviation of 3.85% calculated at this point, while in the internal node it was only 1.54%.

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What is your added value with respect to other techniques?

Clearly focus each of its features

Clearly explain the problem to be solved and the central hypothesis.

Explanation of sections Article.

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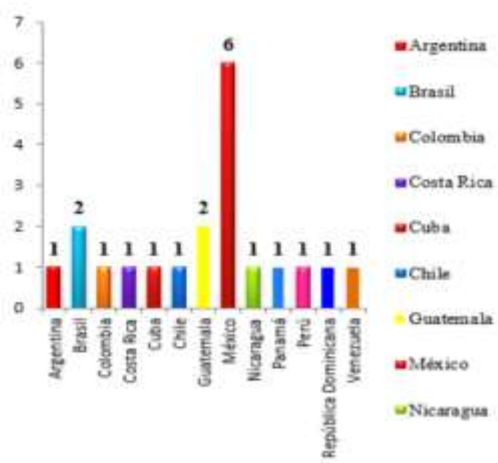

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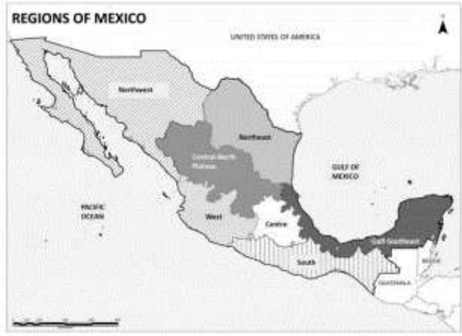

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Products	Industry	Chocolate Business
Food and beverage provision services	Processed food	
	Cultural tourism	Commercial chocolate (national and international brands)
Cultural Services	Agroindustry	Museums of chocolate

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