Comparative analysis of statistical methods for estimating Weibull parameters

Análisis comparativo de los métodos estadísticos usados para estimar los parámetros de Weibull

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Received June 27, 2018; Accepted September 18, 2018

Abstract

This paper describes and compares four of the most used statistical methods of the specialized literature for estimating Weibull parameters, the scale parameter *c* and the form factor *k*, which are widely used in wind energy applications for approximating the random distribution of wind speeds for a given location. The particular characteristic of this paperis doing a comparative analysis to determine the statistical method that offers the highest precision, specifically, for several sites with high wind potential in the region of the Isthmus of Tehuantepec. To carry out this study, it is obtained, firstly, the annual wind measurements of three anemometric stations located in the municipalities of Ixtepec, Juchitan de Zaragoza and Santo Domingo Tehuantepec at the Oaxaca state. Next, a statistical analysis of the wind data in Matlab is made and the parameters *c* and *k* for the methods of Variance, Justus, Moments and Least Squares are determined. Finally, based on the percentage relative error criterion, a comparative analysis of the results obtained from each of the statistical methods studied is carried out and the most accurate method is determined for being used at the wind conditions of the Isthmus of Tehuantepec.

Weibull distribution, Statistical methods, Wind energy

Resumen

En este artículo se describen y se comparan los cuatro métodos estadísticos más utilizados de la literatura especializada para la estimación de los parámetros de Weibull, el parámetro *c* y el factor forma *k*, los cuales se utilizan ampliamente en la energía eólica para aproximar la distribución aleatoria de las velocidades del viento para un emplazamiento determinado. La característica particular de este artículo es la de hacer un análisis comparativo para determinar el método estadístico que ofrece la mayor precisión para varios emplazamientos en la región del Istmo de Tehuantepec. Para llevar a cabo este estudio se obtuvieron, primeramente, las mediciones anuales del viento de tres estaciones anemométricas ubicadas en los municipios de Ixtepec, Juchitán de Zaragoza y Santo Domingo Tehuantepec del estado de Oaxaca. Posteriormente, se hace un análisis estadístico de los datos del viento en Matlab y se determinan los parámetros *c* y *k* para los métodos de Varianza, Justus, Momentos y Mínimos Cuadrados. Finalmente, con base al criterio del error relativo porcentual se realiza el análisis comparativo de los resultados obtenidos con cada uno de los métodos estadísticos estudiados y se determina el método más preciso para las condiciones de viento del istmo de Tehuantepec.

Distribución de Weibull, Métodos estadísticos, Energía eólica

Citation: RODRÍGUEZ-SOLANO, Benjayil, VASQUEZ-CASTELLANOS, Rolando, IRACHETA-CORTEZ, Reynaldo and DORREGO-PORTELA, José Rafael. Comparative analysis of statistical methods for estimating Weibull parameters. Journal-Mathematical and Quantitative Methods. 2018. 2-3: 18-28.

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Introduction

In recent years, wind energy has positioned itself as one of the renewable energy sources with the highest growth in Mexico and in the world (GWEC, 2016). This is mainly due to the promotion of energy policies for the generation of electricity from clean energy sources, the deregulation of the electricity market, environmental objectives, incentives and technical maturity (Dennis et al., 2012; Wais, 2017). During the year 2017, the installed capacity of wind energy in Mexico reached 3,942.22 MW, representing 5.32% of all the installed generation capacity of the country and reported a growth of more than 20% over the previous year (Renewable Report, 2017). The annual production of energy in a wind farm depends on the average speed and standard deviation of the wind speeds, as well as the characteristics of the location. Therefore, for the development of a wind project it is necessary to make measurements of the wind resource in the selected site to obtain the annual distribution of average speeds and the predominant directions of the wind (Haiyan, 2017). The complexity to characterize average wind speeds is due to the fact that it is an intermittent source of energy that is continuously changing during the day and season and even to some extent from year to year. However, wind variability can be accurately characterized in terms of a probability distribution function. Subsequently, with the results of the analysis of the wind resource it is possible to reliably determine the annual energy production, as well as the estimation of losses and uncertainties inherent to the wind variability.

In the specialized literature it is mentioned that the Weibull distribution is the most appropriate and the most used to adjust the distributions of the average wind speed (Carta et al., 2009; Wais, 2017). However, it is important to note that the accuracy of the adjustment depends greatly on the estimation method and the type of location. In this article, the data of annual wind measurements of three anemometric stations located in the region of the Isthmus of Tehuantepec, Oaxaca are analyzed with the statistical methods of Variance, Justus, Moments and Least Squares to determine the energy potential of the wind through the Weibull parameters, the dimensionless parameter of scale c and the form factor k in m / s.

Subsequently, the results obtained with the four statistical methods are compared graphically and analytically, taking Windographer commercial software as reference for comparison. Finally, the most precise method for the characteristics of the wind in the Isthmus of Tehuantepec region is determined.

Description of the study sites

The region of the Isthmus of Tehuantepec, Oaxaca, located southeast of the Mexican Republic, is the point of interest of this work, because it is an area of high potential in wind resource (Elliott et al., 2004). In this article the wind data are analyzed in three anemometric stations located in the municipalities of Ixtepec and Santo Domingo Tehuantepec, as well as in the town of La Venta belonging to the municipality of Juchitán de Zaragoza, as shown in Figure 1.

Geographic coordinates and other relevant information such as elevation above sea level, the height of the measurement sensors, the start and end dates of measurements and the times between measurements for each of the three anemometric stations are described in Table 1**.**

Data	Ixtepec	La Venta	UNISTMO
Latitude	16° 34’ 00’’ N	94° 49’ 04’’ N	16° 17’’ 93’’ N
Length	95° 06’ 00’’ O	16° 34’’ 11’’O	95° 14’ 93’’ O
Elevation M.S.N.M.	40.7	20	43
Height of sensors (m)	20 y 40	15 y 32	10
Data to analyze	52560	52560	451604
Start of measurements	01/01/2007	01/01/2016	01/01/2015
Completion of measurements	31/12/2007	31/12/2016	01/01/2016
Time between measurements (min)	10	10	1

Table 1 General data of the stations
Source: Self Made

ISSN-2531-2979

Figure 1 Location of the three anemometric stations for the analysis of the wind resource
Source: Google Earth Pro

Figure 2 shows the wind atlas of the state of Oaxaca Mexico, where the characteristics of the wind and the distribution of the wind resource can be observed. In the wind atlas, prospective areas for the use of wind technologies can be identified for applications that include electricity generation on a commercial scale, energy for populations and off-grid wind energy.

The region of the Isthmus of Tehuantepec is the region of the state of Oaxaca that concentrates an excellent wind resource with class 5 and / or higher classification, which is equivalent to a wind potential greater than 500 W / m2 or average wind speeds greater than 7.3 m / s at a height of 50 m above sea level.

The areas with the highest wind potential in the region of the Isthmus of Tehuantepec are class 7 and are located near the towns of La Ventosa, La Venta and La Mata. However, for the analysis of the wind resource should also consider the orography of the site in the presence of rugged terrain (Elliott *et al.* 2004).

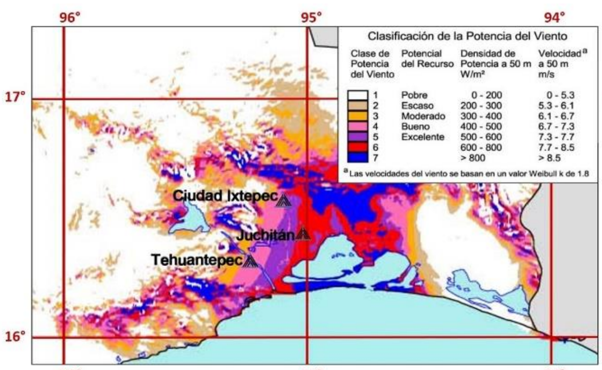


Figure 2 Classification of wind power
Source: Elliott et al. 2004

Wind speed statistics

The random nature of the wind speed can be described analytically by means of statistical methods. Next, some concepts of probability and statistics used for the characterization of the wind are described.

A. Average wind speed

This parameter represents the average wind speeds over a specific time interval. In the wind resource analysis, this parameter is used to quantify the wind potential at a given location. For the calculation of the average wind speed the following equation is used:

$$v = \frac{1}{n}\sum_{i=1}^{n} v_i \qquad (1)$$

Where n is the size of the sample or the number of measured values, and, vi are the instantaneous wind speeds in m / s.

B. Variance

The variance is de fi ned as the quantification of the distances of the data with respect to the value of the mean, that is, it is the recording of the measurement data for the wind speeds with respect to a nominal velocity presented by the study site. , therefore, the following mathematical equation is implemented.

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(v_i - v)^2 \qquad (2)$$

Where n is the number of available data and, $(v_i\text{-}v)^2$ is the difference of the average speed and the speeds recorded in m / s.

C. Standard deviation

The standard deviation is a measure of dispersion of the burst velocities around the mean, and indicates that the standard error of the wind is with respect to the prevailing wind velocity at the site with respect to the square root of the velocity fl uctuations of the wind (Rocha et al., 2012). It is determined by extracting the square root from the variance, obtaining the following equation:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(v_i - v)^2} \quad . \tag{3}$$

Numerical methods for the calculation of Weibull parameters

For the statistical calculation of the wind potential at a given location, knowledge of the distribution of wind speeds is required. To make this calculation it is necessary to have records of time series of wind speeds of at least one year. Such records are known as wind data. In the specialized literature, different models of distribution functions are described to adjust wind speeds over a period of time (Carta et al., 2009; Conradsen et al., 1984; Haiyan J. et al., 2017; Kaogaet al., 2014; Rocha et al., 2012; Saleh et al., 2012; Udoakahet al., 2017). However, the Weibull function of two parameters has been for many years the most widely accepted technique in wind energy applications (Wais, 2017). The Weibull distribution can be described as a function of wind probability density f (v) and a cumulative distribution function F (v), determined by the following equations:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right], \quad v \geq 0\,; \tag{4}$$

$$F(v) = 1 - \exp\left(\frac{v}{c}\right)^{k} \tag{5}$$

Where c and k are the Weibull parameters and v is the wind speed in m / s. To estimate the Weibull parameters, the following four statistical methods are described.

A. Method of Variance

This method considers the values of average velocity in (1) and the standard deviation in (3) to obtain the coefficient of variation of the wind. (Carrillo et al., 2013)

$$I = \left(\frac{\sigma}{v}\right) * 100. \tag{6}$$

From the calculation of the coefficient of variation of the wind (6) we proceed to determine the rank to which it belongs, according to the values shown in Table 2.

No.	Range of Variation I (%)	Type of Variance
1	$0 < I \leq 33$	Low
2	$33 < I \leq 66$	Medium
3	$66 < I \leq 100$	High

Table 2 Range of coefficient of variation
Source: Carrillo et al., 2013

Depending on the range of variation of I, one of the three equations shown below is used to determine the form factor k.

Low range

$$k = 1.05\left(\sqrt{v}\right) \tag{7}$$

Middle range

$$k = 0.94\left(\sqrt{v}\right) \tag{8}$$

High rank

$$k = 0.83\left(\sqrt{v}\right) \tag{9}$$

Obtaining the form factor k, proceeds to obtain the scale factor c, through the following equation:

$$c = v\left(0.568 + \frac{0.433}{k}\right)^{\frac{1}{k}} \tag{10}$$

B. Justus method

Also called empirical method (Bagiorgas et al., 2012), it is considered a special case of the Moments method, where the Weibull parameters are determined using the mean velocity v and the standard deviation σ:

$$k = \left(\frac{\sigma}{v}\right)^{-1.086} \tag{11}$$

To calculate the scale factor with this method, just like the Variance method, the expression (10) is used.

C. Moments Method

With this technique, the parameters k and c are determined by the following equations (Gautam & Gautamrao, 2015):

$$k = \left(\frac{0.9874}{\frac{\sigma}{v}}\right)^{1.983} \tag{12}$$

ISSN-2531-2979

$$c = \left(\frac{v}{\Gamma\left(1 + \frac{1}{k}\right)} \right) \tag{13}$$

Where Γ is the Gamma function.

D. Least Squares Method

With the Least Squares method, we can find the Weibull cyka parameters from the linearization of the cumulative distribution function F (v) described in (5) to subsequently minimize the sum of the squared error between the linearized ideal curve and the points of current data provided by the distribution model (Kaplan, 2017). As a first step of the method, equation (5) is linearized as follows

$$\ln[-\ln(1 - F(v))] = k\, ln\,(v) - k\, ln\,(c) \tag{14}$$

To get the shape of the equation of the straight line

$$y = ax + b \tag{15}$$

Where x and y are variables, a is the slope, and b is the intercept of the line on the y axis. In particular,

$$y = ln[-ln(1 - F(v))], \; a = k, \; x = ln\,(v), \; y\; b = -kln\,(c). \tag{16}$$

From this equation are the following parameters

$$a = \frac{\sum_{i=1}^{w} x_i y_i - \frac{\sum_{i=1}^{w} x_i \sum_{i=1}^{w} y_i}{w}}{\sum_{i=1}^{w} x_i^2 - \frac{(\sum_{i=1}^{w} x_i)^2}{w}} \tag{17}$$

So a is equal to

$$a = \frac{\sum_{i=1}^{w}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{w}(x_i - \bar{x})^2}, \tag{17}$$

$$b = \bar{y}_i - a\bar{x}_i = \frac{1}{w}\sum_{i=1}^{w} y_i - \frac{a}{w}\sum_{i=1}^{w} x_i \tag{18}$$

In these equations $\bar{x}$ and $\bar{y}$ are the mean values of x_i and y_i, and w is the total number of available samples. The final results for the Weibull parameters are

$$k = a, \; c = exp^{\left(-\frac{b}{k}\right)}. \tag{19}$$

To improve the approximation by the least squares method in the most common velocity range, a weight coefficient is redefined that depends on the function of the wind probability density to transform equation (19) into

$$a = \frac{\sum_{i=1}^{w} p^2(u_i)(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{w} p^2(u_i)(x_i - \bar{x})^2}. \tag{20}$$

This equation effectively multiplies each value of xi and yi by the probability that xi and yi will occur. In general, this approach guarantees a better fit than the unweighted form of equation (17) (Udoakah & Ikafia, 2017, Bagiorgaset al., 2011).

Performance of energy conversion systems

The available power of the wind that crosses the sweeping area of a wind turbine is given by

$$P_w(v) = \frac{1}{2} A\rho(v)^3 \tag{21}$$

Where $P_w(v)$ is the power associated with the wind speed v in m / s, A is the rotor area in m^2 and ρ is the air density in kg / m^3. The relationship between the mechanical power (P_{mec}) of the wind turbine input and the available power of the wind (P_w) is given by the power coefficient

$$C_p = \frac{P_{mec}(v)}{P_w(v)} \tag{22}$$

It depends on the design of the blade, the angle of inclination and the relationship in the speed of the rotor and the wind speed. Its maximum theoretical value, known as the Betz limit, is 16/27 ≈ 0.593. However, this value may not be achievable in a real wind turbine; Typically, its maximum value is approximately 0.5.

The energy potential of a site, expressed in W / m^2, is obtained with

$$P(v) = \frac{p}{A} = \frac{1}{2}\rho(v)^3 = \frac{1}{2}\rho \sum_{i=1}^{w} f(v_i) v_i^3 \tag{23}$$

Where $f(v_i)$ is called the wind power density distribution and represents the probability that the average wind speed is in the range of v_i±0.5 m/s.

ISSN-2531-2979

When considering the probability distribution function of Weibull, the average power density of the wind is given by

$$\bar{P}_w(v) = \frac{P_w}{A} = \frac{1}{2}\rho(\bar{v})^3 \frac{\Gamma\left(1+\frac{3}{k}\right)}{2[\Gamma\left(1+\frac{1}{k}\right)]^3}. \quad (24)$$

The annual energy production at speed vi is the power in the wind multiplied by the fraction of time multiplied by the number of hours in the year. If v_{me} is the wind speed that produces the greatest amount of energy compared to any other wind speed. Therefore, the maximum energy obtained for any wind speed is

$$W_{max} = \frac{1}{2}\rho A(v_{me}^3)^3 f(v_i)(8760) \quad (25)$$

where

$$v_{me} = c\left(\frac{k+2}{k}\right)^{1/k}. \quad (26)$$

For the design of wind turbines it must be considered that the speed with the maximum energy content is included in its best operating range of wind speeds. Some applications require that the turbine be designed at a nominal wind speed equal to the maximum energy wind speed.

1. Analysis of the results

First, to perform the comparative study of the four statistical methods; Variance, Justus, Moments and Least Squares, we obtained the annual measurements of the wind data of three anemometric stations located in the city of Ixtepec, La Venta and the Campus of the Universidad del Istmo and belonging to the municipalities of Ixtepec, Juchitán de Zaragoza and Santo Domingo Tehuantepec, respectively.

The information regarding each of the anemometric stations is described in Table 1. The wind statistics are summarized in Table 3. In this table it can be seen that the measurement site with the highest wind potential and with the highest turbulence index According to the norm, it is the location of the sale, for measuring heights of 15 and 32 meters, followed by the city of Ixtepec, at 20 and 40 meters. The campus of the Universidad del Istmo is the site with the lowest wind potential and the lowest turbulence index.

Station	Height (m)	Average speed (m/s)	Standard deviation (m/s)
Ixtepec	20	6.24	3.51
	40	7.33	4.02
La Venta	15	9.58	5.45
	32	10.769	5.865
UNISTMO	10	2.85	2.06

Table 3 Wind statistics of the three measurement sites
Source: Self Made

The results corresponding to the form factor k for the methods of Variance, Justus, Moments, Least Squares and the commercial software Windographer and for each one of the different measurement heights are shown in Tables 4 a) and 4 b). The results of this table were calculated from the statistical wind data shown in Table 3**.**

Station	Height	Variance	Justus	Moments
Ixtepec	20	2.348	1.866	1.853
	40	2.546	1.923	1.91
La Venta	15	2.9	1.846	1.833
	32	3.084	1.934	1.922
UNISTMO	10	1.41	1.42	1.41

Table 4 a). Results obtained for the form factor k
Source: Self Made

Estación	Altura	Mínimos Cuadrados	Windographer
Ixtepec	20	1.559	1.575
	40	1.699	1.769
La Venta	15	1.513	1.61
	32	1.625	1.81
UNISTMO	10	1.43	1.21

Table 4 b). Results obtained for the form factor k
Source: Self Made

With the corresponding values of the form factor k for each method, we proceeded to calculate the scale factor c with equations (10), (13) and (19), and whose results are shown in Tables 5 a) and 5 b)**.**

Station	Height	Varianza	Justus	Momentos
Ixtepec	20	5.52	5.535	7.023
	40	6.51	6.502	8.267
La Venta	15	10.75	10.801	10.798
	32	12.04	12.194	12.147
UNISTMO	10	1.44	2.61	2.6

Table 5 a). Results obtained for the scale factor c
Source: Self Made

ISSN-2531-2979

Station	Height	Least Squares	Windographer
Ixtepec	20	6.554	6.81
	40	7.044	8.15
La Venta	15	10.64	10.53
	32	12.037	12.02
UNISTMO	10	2.56	3.07

Table 5 b). Results obtained for the scale factor c
Source: Self Made

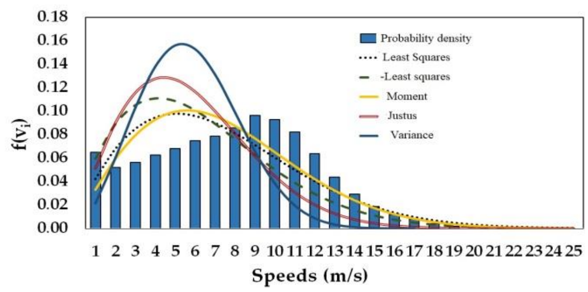
From the Weibull cyk parameters, shown in Tables 4 a) and 4 b) and Tables 5 a) and 5 b), the estimated average speed probability density functions are calculated for each of the four statistical methods and commercial software Windographer. Figures 3a-b, 4a-b and 5 show the results of the probability distributions for the average velocity in the anemometric stations located in the city of Ixtepec, the town of La Venta and the campus of the UNISTMO in Tehuantepec. It can be seen that the three sites have very favorable conditions for the installation of wind turbines. The maximum frequency ranges of the average winds for the city of Ixtepec are between 7 and 11 m / s, for La Venta they are between 12 and 17 m / s and finally, for the UNISTMO campus they are between 1 and 5 m / s. To compare the graphical results, the criterion of the absolute percentage percentage error is used as

$$\%E_{abs} = \frac{100}{N}\left(\sum_{i=1}^{N}\left(p(v_i) - f(v_i)\right)\right) \qquad (27)$$

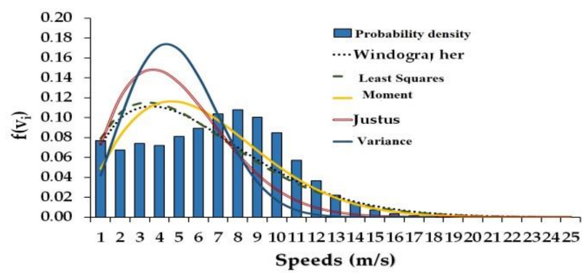
to compare the accuracy of the statistical methods used to calculate the Weibull parameters. In this equation N is the number of observations, $p(v_i)$ is the probability of the wind speeds calculated from the measurements and $f(v_i)$ is an estimated value of the probability for the wind speed v_i. Tables 6 a) and 6 b) show the summary of the average absolute percentage errors obtained with each of the statistical methods analyzed and with the commercial software Windographer, and for each of the three sites at different heights.

From the results shown in Figures 3a and 3b it can be seen that the Moments method presented the minimum absolute percentage errors with 1.22% and 1.16% for 20 and 32 m height, respectively. The percentage errors obtained with the methods of Least Squares and Windographer turned out to be very similar to those of the Moments method.

In contrast, the methods of the Variance and Justus were the most inaccurate to adjust the wind data. In Figures 3a and 3b it is clearly observed that the use of these variance and moment methods can overestimate the wind potential available in the Isthmus of Tehuantepec region. The results of the wind distributions for the locality of the Sale in Figures 4a and 4b show that the Moments and Justus methods were the most accurate, that is, they had minimum percentage errors of around 1.20% for a height of 15 m.

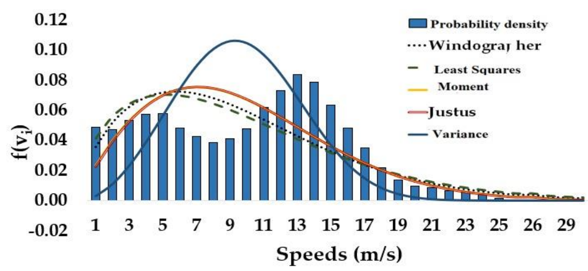


a)

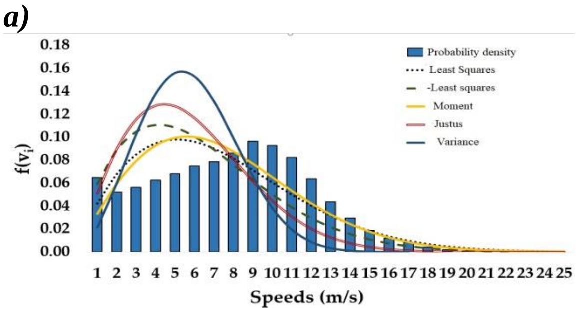


b)

Figure 3: *Distribution of wind speeds in Cd. Ixtepec. a) 20 m high, b) 40 m high.*
Source: Self Made



a)



b)

Figure 4 Distribution of average wind speeds in the town of La Venta. a) 15 m high, b) 32 m high
Source: Self Made

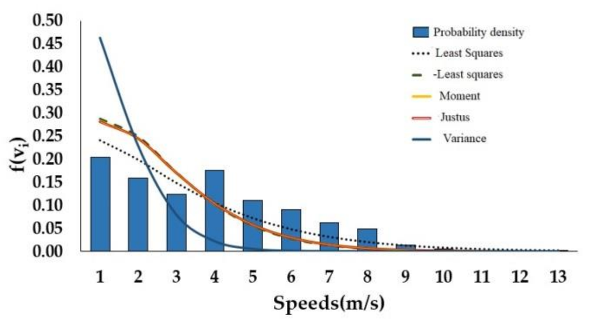


Figure 5 Distribution of wind speeds at the Tehuantepec Campus of the UNISTMO at 10 m height
Source: Self Made

Station	Height	Variance (%)	Justus (%)	Moments (%)
Ixtepec	20	2.748	2.238	1.222
	40	2.892	2.286	1.163
La Venta	15	2.012	1.202	1.199
	32	1.984	1.137	1.140
UNISTMO	10	6.13	2.666	2.663

Table 6 a) Average absolute percentage error of Weibull distributions
Source: Self Made

Station	Height	Least Squares (%)	Windographer (%)
Ixtepec	20	1.422	1.338
	40	1.733	1.246
La Venta	15	1.272	1.263
	32	1.167	1.125
UNISTMO	10	2.9223	2.500

Table 6 b) Average absolute percentage error of Weibull distributions
Source: Self Made

In Table 7, a summary of the power density estimate for the three sites studied is shown. The calculation of the power density was obtained with the methods of moments and least squares since they were the ones that reported the minimum percentage absolute errors for all the study cases. Additionally, to facilitate the comparison of the wind potential in the three sites, wind data from La Venta and UNISTMO were extrapolated to 20 m and 40 m high.

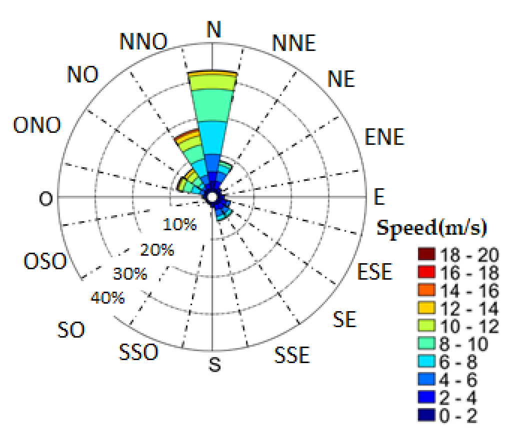
From Table 7 it is possible to observe the locality of La Venta where the greatest wind potential is available with values in the range of 1665.743 to 2033.864 W / m2 at a height of 40 m. Then, in the city of Ixtepec, a wind potential was estimated in the range of 308,068 and 526,678 W / m2 for heights of 20 and 40 m, respectively.

The campus of the UNISTMO in Tehuantepec recorded an estimate in the range of 47,197 to 65,951 W / m2 for 20 and 40 m in height.

Site	Height (m)	Power density (W / m2)	Method
Ciudad Ixtepec	20	382.641	Least Squares
	40	526.678	
	20	308.068	Moments
	40	483.14	
La venta	20	1,645.25	Least Squares
	40	2,033.86	
	20	1284.36	Moments
	40	1665.743	
UNISTMO Campus Tehuantepec	20	47.197	Least Squares
	40	48.269	
	20	64.478	Moments
	40	65.941	

Table 7 Power density in the 3 sites
Source: Self Made

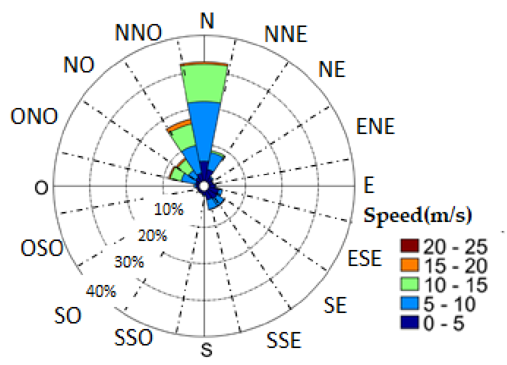
In the analysis of the wind resource it is essential to determine the distribution of the direction of the prevailing winds. In practice, the most usual way to represent the distribution of the wind direction is the wind rose, which indicates the percentage of time in which the wind blows from different directions, that is, it has the purpose of demonstrating the information on the distributions of wind speeds and the frequency of variation of the directions of the same. In Figures 6 a) and 6 b), the wind rose is shown in 16 sectors of the Ixtepec station at 20 and 40 meters respectively, in which it can be seen that the direction of greatest wind potential would be the North (N), which corresponds to a speed of 18 m / s up to 20 m / s, with a frequency of 32%.



a)

Figure 6 a) Annual wind rose of Ixtepec at 20 m height
Source: Self Made

ISSN-2531-2979



b)

Figure 6 b) Annual wind rose of Ixtepec at 40 m height
Source: Self Made

In Figures 7 a) and 7 b) the wind roses of the Sale area at 15 and 32 m high are observed. It can be seen that the predominant wind direction is in the Northwest direction (NNO) with 65% of the wind frequency in the range of 0 to 30 m / s. Figure 8, on the other hand, shows the wind rose at the Tehuantepec Campus of UNISTMO at a height of 10 m. It can be seen that the predominant wind directions are in the North (N) and Northwest (NE).

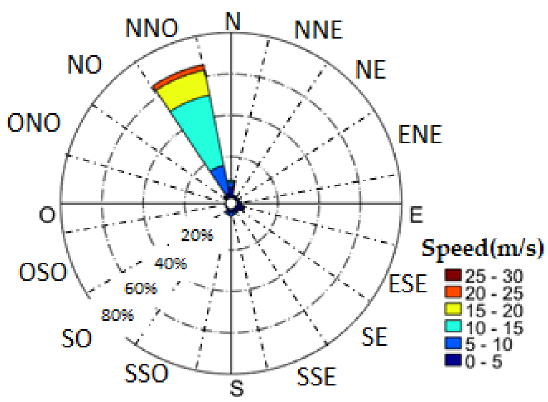


Figure 7 a) Compass rose in La Venta at 15 m height
Source: Self Made

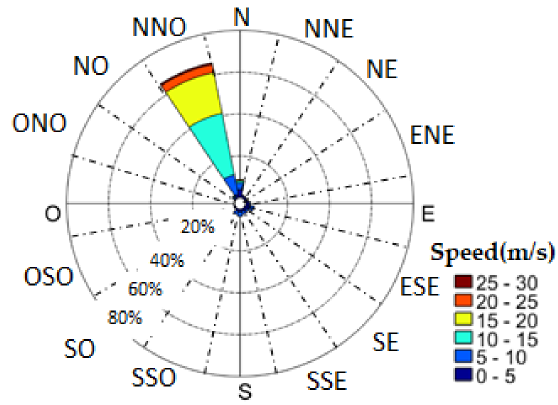


Figure 7 b) Compass rose in La Venta at 32 m height

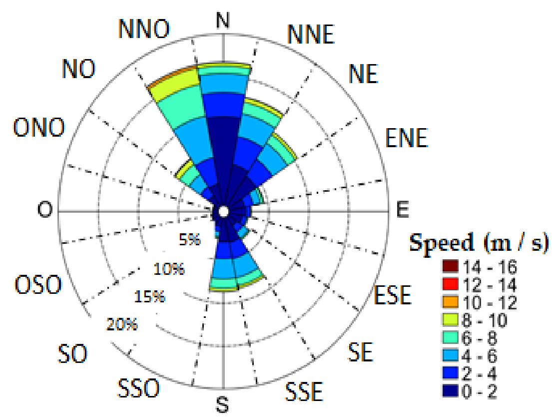


Figure 8 Rosa de los vientos at the Tehuantepec Campus of the UNISTMO at 10 m height
Source: Self Made

Conclusions

The Weibull distribution of two parameters, c and k, is recognized within the sector of the wind industry as the most appropriate model and the most widely used for estimating wind potential at a given site. The accuracy of the estimation depends considerably on the calculation of the parameters of c and *k*. In the specialized literature, many methodologies have been proposed for the calculation of Weibull parameters.

However, these methodologies are only valid for certain site-specific conditions and, therefore, do not guarantee the accuracy of the results. In this investigation, wind resource studies were carried out for three sites with high wind potential in the region of the Isthmus of Tehuantepec, Oaxaca, Mexico. Subsequently, the wind data were characterized with four statistical methods and a comparative analysis was made to determine the most accurate methodology to estimate the wind potential in each of the locations.

From the research carried out, it is concluded that the analysis of the wind resource in a specific area is essential to evaluate the technical and financial feasibility of a wind project. To make the analysis of the wind resource, measurements of at least one year at the site of the site are required. Subsequently, the wind resource should be characterized by means of statistical methods, in the study site, to determine the parameters of Weibull c and k. The precise estimation of these parameters is very important to determine the income during the life of the project.

From the comparisons made, through the graphs of distributions of average wind speed and the absolute average percentage error criterion, it is concluded that with the method of moments the minimum absolute percentage percentage errors are reached for the estimation of the wind resource in the localities of Ixtepec and La Venta, respectively, where the greatest potential is counted in the region of the Isthmus of Tehuantepec. With the Windographer software, the minimum absolute percentage error in the Tehuantepec Campus of the UNISTMO was obtained. In contrast, with the Justus method, a good approximation of the velocity distributions was obtained in La Venta and UNISTMO. In general terms, the methods of moments, least squares and Windrographer offer good results for the estimation of the wind resource in the three sites studied. Finally, the variance method registered the highest percentage absolute average errors for all the study cases and, therefore, it is the most imprecise for the calculation of wind potential in the region of the Isthmus of Tehuantepec.

Acknowledgement

The authors thank the SENER-CONACYT-Energy Sustainability Fund and the CEMIE-Eolic for the resources contributed, as well as the CONACYT Chairs program.

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ISSN-2531-2979