

## Solving a robust parameter problem that includes a qualitative noise variable

### Solución de un problema de parámetros robustos con una variable de ruido cualitativa

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#### Abstract

Response Surface Methodology has been widely used to solve robust parameter problems that include control variables such as quantity of material, temperature, pressure, time, etc. and noise variables difficult or impossible to control such as humidity, room temperature, etc. The noise variables included in the problems so far have been quantitative. However, these noise variables could be qualitative: type of machine, operator, type of supplier, etc. The article proposes a response surface methodology when there is a qualitative noise factor. The first steps of the investigation are explained in detail: construction of the general regression model, assumptions of the model, design of the steps of the methodology, simulations using the methodology and the solution of a case with one quantitative control variable and one qualitative noise variable, in addition to results that demonstrate the effectiveness of the methodology.

**Response surface, Qualitative noise variables, Dual response surface methodology**

#### Resumen

La Metodología de Superficie de Respuesta ha sido ampliamente utilizada para resolver problemas de parámetros robustos que incluyan variables de control como cantidad de material, temperatura, presión, tiempo etc. y variables de ruido difíciles o imposibles de controlar como humedad, temperatura ambiente, etc. Las variables de ruido incluidas en los problemas hasta ahora han sido cuantitativas. Sin embargo, éstas podrían llegar a ser cualitativas: tipo de máquina, operador, tipo de proveedor, etc. El presente artículo plantea una metodología de superficie de respuesta cuando se tiene un factor de ruido cualitativo. Se explica de manera detallada los primeros pasos de la investigación: construcción del modelo general de regresión, supuestos del modelo, diseño de los pasos de la metodología, simulaciones empleando la metodología y la solución de un caso con una variable de control cuantitativa y una variable de ruido cualitativa, además de resultados que demuestran la efectividad de la metodología.

**Superficie de respuesta, Variables de ruido cualitativas, Metodología de superficie de respuesta dual**

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## Introduction

The response surface methodology is a set of statistical and mathematical techniques used in the analysis and modeling of problems in which a study variable is affected by other variables.

The origins of the response surface methodology (MSR) refer to the work of Box and Wilson (1951), but it has been during the last 20 years that this methodology has achieved considerable development, both in theoretical aspects and in applications in scenarios real.

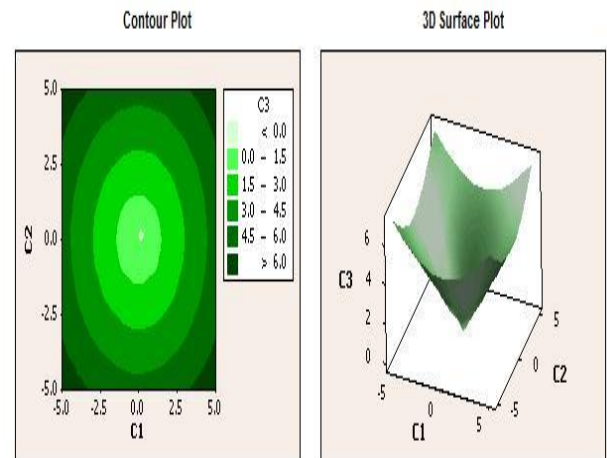
Myers and Montgomery (2009) establish that the MSR provides well-established statistical techniques that can be used to implement the robust parameter design proposed by Taguchi (1986) and overcome its limitations.

The main objective of robust design is to find the optimal level of the controllable factors in a process or product in order that noise or non-controllable factors do not affect the process. Nowadays, statistical applications for process optimization are a useful tool used by most companies to optimize the average of their processes and decrease the variability.

Previously, problems of robust factors have been solved using the response surface method with quantitative noise factors, however there is no record of solved problems where the noise factors are qualitative.

The ultimate goal of the MSR is to determine the optimum operating conditions of the system or to determine a region of the factor space in which the operating requirements are met (Montgomery, 2005).

To perform the response analysis, many authors have proposed the use of separate models to measure the mean and variance. This method, introduced by Myers and Carter (1973), it is called a dual response surface where the mean can be optimized and simultaneously minimizing the variance as shown in figure 1.



**Figure 1** Graphic de contornos y de superficie de respuesta

Source: Minitab

Considering the second-degree response surface model that includes a control factor ( $x$ ) and one of qualitative noise ( $z$ ) results in equation 1.

$$y = \beta_0 + \beta_1x + \beta_{11}x^2 + \delta_1z + \delta_{11}xz + \varepsilon \quad (1)$$

Where the betas represent the regression coefficients of the effects of the control factor, the deltas represent the coefficients of the noise factor and the interaction between the control factor and the noise and the epsilon the random error of the model.

The noise factor is a random variable whose probability distribution, its expected value and its variance are not usually known. Applying the operator of the expected value and the variance, we obtain equations 2 and 3.

$$E(y) = \beta_0 + \beta_1x + \beta_{11}x^2 + \delta_1E(z) + \delta_{11}xE(z) \quad (2)$$

$$V(y) = (\delta_1 + \delta_{11}x)^2V(z) + \sigma^2 \quad (3)$$

Since the expected value and the variance of the noise variable  $z$  are not known, they have to be estimated from a representative sample of their possible values.

The main objective of the research presented in this article is to develop a methodology that allows to optimize the mean and the variance simultaneously using the dual response surface method in problems with robust parameters that contain qualitative noise factors.

**Development**

The steps that will be followed to define the dual response surface methodology with qualitative noise variables (MSRDVC) are:

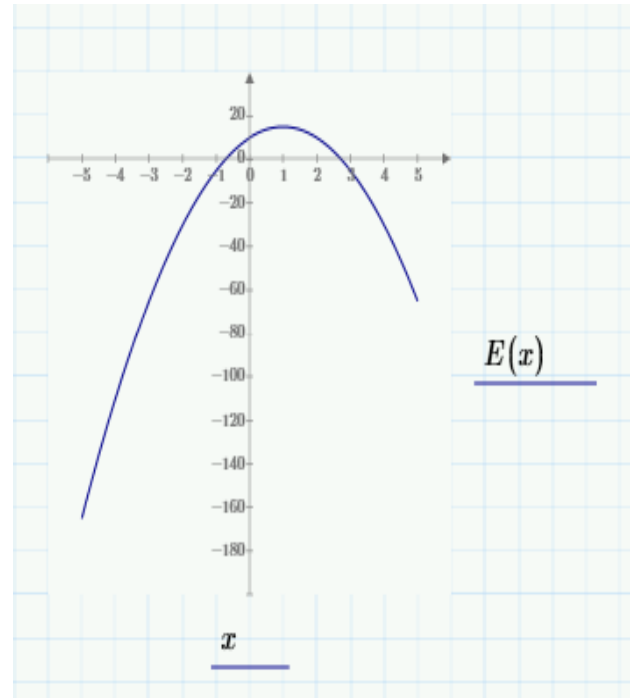
1. Define models of the response whose optimal solution can be known analytically.
2. Use the models to simulate the response in the experimental runs according to an appropriate experimental design.
3. Identify the probability distribution of qualitative noise variables.
4. Obtain the solution that simultaneously optimizes the mean and the variance.
5. Compare the solution obtained with the MSRDVC methodology with the solution obtained analytically to verify the effectiveness of the MSRDVC methodology.
6. Validate the MSRDVC methodology in a real case.

**Methodology**

A case was analyzed that includes a quantitative control variable and a qualitative noise variable. The general model that was used is shown in equation 4 to describe the behavior of the response whose mean or expected value has a maximum of 15 at  $x = 1$  as can be seen in figure 2 graphically using Matlab® software.

$$y(x, z) = 10 + 10x - 5x^2 - 5z + 5x \quad (4)$$

These values of beta, delta and epsilon of the model that represent the coefficients of the control factor and noise respectively as well as the estimator of the random error of the model are obtained analytically by formulating an equation that contains a maximum or a minimum to be able to use it with function of test, and will change when this methodology is used in other cases that will be simulated in the future of the investigation.



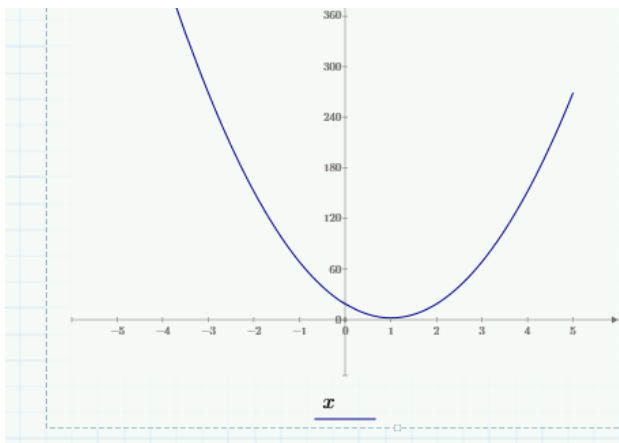
**Figure 2** Test Model for Case 1  
Source: Own MathCad software

Table 2 shows the modified composite central design (DCCM) of the simulation of the response for each corresponding experimental run. Its three parts are the factorial 22, the respective: axial points of the control factor and five central points. To simulate the response, a random number is generated using Excel® software in order to generate a normally distributed random variable with zero mean and a standard deviation of 10% of the optimal average response. This value of 10% was established considering that most of the random variation of the response is due to the noise factor. Therefore, the error must be relatively small. In such a way that the answer  $y$  was generated using equation 6 and adding the simulated random error. Table 1 also shows the mean and standard deviation calculated for the random error.

CCM design					
X	Z	Random number	Standardized Variable	Random Error	Y
-1	-1	0,93646	1,525715869	2,288574	7,2885738
1	-1	0,00291	-2,75779884	-4,1367	10,8633017
-1	1	0,510875	0,027262206	0,040893	-14,9591067
1	1	0,21197	-0,79960604	-1,19941	13,8005909
-1,41421	0	0,924493	1,435956139	2,153934	-11,9881154
1,41421	0	0,428414	-0,18041399	-0,27062	13,8715294
0	0	0,185245	-0,89555573	-1,34334	8,65666405
0	0	0,829078	0,950529528	1,425794	11,4257943
0	0	0,519708	0,049421135	0,074132	10,0741317
0	0	0,809583	0,876359828	1,31454	11,3145397
0	0	0,540677	0,102138419	0,153208	10,1532076

**Table 1** Composite Central Design to Simulate the Response

After obtaining by means of the simulation the answer corresponding to each experimental run of the experimental DCCM design, the response is introduced in the Minitab® software to obtain a linear regression model. We proceed to generate using Minitab® the linear regression model that fits the data obtained from the response. Then, from this general model, the response surfaces are obtained for both the mean and the variance. The graphic of the response surface of the mean was already shown in figure 2 and the graphic of the response surface of the variance is shown in figure 3. In this case, since there is only one control variable, these graphics instead of being surface they are linear graphics, the variance has its minimum at  $x = 1$  with a value of 2.25.



**Figure 3** Graphic of variance  
Source: Own MathCad software

The next step is to identify the distribution to which the data of the noise factors best fit, for our case it was assumed that the behavior of the noise factor would be a discrete uniform probability distribution. For this, the simulation of the noise factors was carried out using the Monte Carlo simulation method and thus be able to generate the levels of the factor and be able to calculate the mean and variance of the qualitative noise factor and be able to have complete the mathematical model proposed. Figure 4 shows the table where the simulation of the values that represent the noise factor levels using Excel® software was shown.

In the first part we can observe the simulation of the noise factor generating a random number between 0 and 1 and then assign three possible different levels for the factor, level -1 if the number is between 0 and .33, level 0 if it is between 0.34 and .66 and finally level 1 if it is between .67 and 1.

It also includes a summary of the mean, standard deviation and variance of the noise factor.

Simulation of qualitative noise variable			
Sample number	Random number		Value of z
1	0,120617	0,42694813	0
2	0,5224727	0,7283256	1
3	0,21349635	0,62383019	0
4	0,05295553	0,27735833	-1
5	0,0772412	0,02948244	-1
6	0,28300435	0,70522494	1
7	0,77207493	0,97926981	1
8	0,36246082	0,74037089	1
9	0,04774751	0,74587227	1
10	0,66399103	0,91091062	1
11	0,3144666	0,8610971	1
12	0,39655564	0,02368823	-1
13	0,5042887	0,4985807	0
14	0,67841467	0,60313573	0
15	0,29274729	0,16131825	-1

Average	0,01
Deviation	0,30462928
Standard	
Variance	0,73727273

**Figure 4** Simulation of the Qualitative Noise Factor  
Source: Own Excel software

### Results

Once the general model has been obtained in the Minitab® software and the mean and variance of the noise factor, Matlab® mathematical software is used to optimize the first and second degree models obtained when doing the dual response surface analysis where the equation of the mean and the variance in its general form is shown in equation 5 and 6 respectively.

$$E(y) = 10 + 10x - 5x^2 - 5E(z) + 5xE(z) \quad (5)$$

$$V(y) = \sigma^2 + (-5 + 5x)^2 * V(z) \quad (6)$$

In addition it is important to calculate the simultaneous criterion and the optimal ideal weight, the simultaneous criterion allows to optimize together the mean and variance in the model for each of the fifteen simulations and at the end of the totals, the equation of this simultaneous criterion is shown in the equation 7. Also an example of one of the fifteen equations is shown in equation 8 for the mean and in equation 9 for the variance.

$$y(simultaneo) = E(y) - pesoV(y) \quad (7)$$

$$y = 10.63 + 9.93x - 4.04z - 5.95x^2 + 5.59xz \quad (8)$$

$$v(y) = (-4.044 + 5.594x)^2 * 6812 + 2.25 \quad (9)$$

The results of the first simulation with a control factor and a qualitative noise factor with a behavior of a uniform distribution are shown in table 2, where the results of 15 simulation runs are shown.

The first two columns include the mean of the noise factor  $E(z)$  and simulated variance  $V(z)$  and in the rest the maximum value of the mean and the minimum value of the variance are shown when optimized independently.

$z$	$Vz$	$E(\max)$	$V(\min)$	max simultaneous	opt weight
0,08	0,6804	14,5662	2,1217	14,5642	1/950
-0,01	0,717	14,1411	1,9997	14,1399	1/1200
0,05	0,6944	16,3876	2,6855	16,3858	1/900
-0,03	0,6758	16,3882	2,6857	16,3862	1/800
0	0,6868	13,286	1,7652	13,2832	1/750
0,13	0,7405	15,537	2,414	15,536	1/1500
0,06	0,562	15,8682	2,518	15,8665	1/1500
0,04	0,6852	16,2304	2,6342	16,229	1/1500
0,07	0,6516	14,7488	2,1753	14,7471	1/900
0,04	0,665	14,7906	2,1876	14,7889	1/850
0,08	0,5793	14,4536	2,0891	14,4518	1/850
0,05	0,6944	15,3928	2,3694	15,3911	1/900
-0,04	0,5842	14,4133	2,0774	14,4117	1/900
0,08	0,7208	14,7649	2,18	14,7632	1/850
0,12	0,5915	15,649	2,4489	15,6467	1/900
0,048	0,661927	15,10785	2,290113	15,10608667	1/1125

**Table 2** Summary of Simulation Results

The proximity of the results of the simulation with the best known response is verified, in order to validate its functionality, the known average response for the first simulation model was 15, if we compare the value of 15.1060 the total percentage error would be .7%, added to an average variance of 2.29 that is realit a low value and compared to the minimum value of the variance that is 2.25 is 1.23%. This would result in the fact that if 15 were the average length of a cable, the qualitative noise factor  $z$  could be estimated and the response optimized, being very close to the ideal response.

## Conclusions

As can be seen in the summary table of results on average, all the results were close enough to the optimal response, which shows results very close to the optimal value of the average of 15 and an average variance of 2.29.

Regarding the results observed in this first simulation using the proposed methodology, encouraging results are observed and in the future, different cases will be simulated with the purpose of verifying that the methodology resolves different cases that may arise in the practice of the processes industrial.

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