Mathematical modeling in STEM programs: The vibrating string

Modelado matemático en programas STEM: La cuerda vibrante

LÓPEZ-REYES, Luis Javier*†, JIMÉNEZ-GUTIÉRREZ, Auria Lucia, LARA-RAMÍREZ, Larisa Elizabeth, COSTILLA-LÓPEZ, Diana

Universidad de Guadalajara / CULagos. Enrique Díaz De León #1144, Lagos de Moreno, Jalisco; México.

ID 1st Author: *Luis Javier, López-Reyes* / **ORC ID** 0000-0002-3534-1070, **CVU CONACYT ID**: 235848

ID 1st Coauthor: *Auria Lucia, Jiménez-Gutiérrez* / **ORC ID**: 0000-0002-7576-0122, **CONACYT ID**: 946766

ID 2nd Coauthor: *Larisa Elizabeth, Lara-Ramírez /* **ORC ID**: 0000-0002-5564-6808, **CVU CONACYT ID**: 239391

ID 3rd Coauthor: *Diana, Costilla-López /* **ORC ID**: / 0000-0002-3667-6139, **CVU CONACYT-ID**: 239398

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Abstract

In Mexico, the 21st century During STEM programs Have received a strong boost from the government and the labor sector. This requires a commitment from Educational Institutions and Represents an academic challenge for teachers in the design of learning environments. Particularly, in the teaching and learning of mathematics one of the educational practices orients the student That better in a meaningful way towards learning is mathematical modeling and is more solid if the modeling is of actual systems found in His professional practice or in daily life. This document shows, on the one hand, The Importance of Systematically Including experimental research Both theoretical and actual of systems in STEM programs, as part of the academic training of future professionals; on the other hand, the result of a mathematical modeling of real physical system,

Mathematical modeling. Meaningful learning.Vibrating string. STEM

Resumen

En México, durante el siglo XXI los programas STEM han recibido un fuerte impulso desde el gobierno y el sector laboral. Esto requiere de un compromiso de las instituciones educativas y representa un reto académico para los profesores en el diseño de ambientes propicios de aprendizaje. En particular, en la enseñanza y aprendizaje de las matemáticas una de las prácticas educativas que orienta al estudiante de mejor manera hacia un aprendizaje significativo es la modelación matemática, y es más sólido si el modelado es de sistemas reales que encuentre en su ejercicio profesional o en la vida cotidiana. En este documento se muestra, por un lado, la importancia de incluir de manera sistemática la investigación tanto teórica como experimental de sistemas reales en programas STEM, como parte de la formación académica de futuros profesionistas; por otro lado, el resultado de una modelación matemática de un sistema físico real, la cuerda vibrante, dado que actualmente se observa en importantes aplicaciones tecnológicas.

Modelación matemática. Aprendizaje significativo. Cuerda vibrante. STEM

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* Correspondence to Author (email: javierlopez@culagos.udg.mx)
† Researcher contributing first author.

Introduction

Mathematics education scientific studies have confirmed the importance of mathematical modeling to achieve the goal of meaningful learning in students of educational programs Profile of Science, Technology, Engineering and Mathematics (STEM acronym in English); very recently in an investigation so it was concluded that:

Mathematical modeling as a teaching resource inside and outside the classroom, applied to every situation of daily life [...] is a good choice for students to discuss and express their ideas, perspectives and personal knowledge. Fosters interest in learning mathematics through its connection with the situations in context, and allows students to participate actively and dynamically in their learning processes.(Pantoja Rangel, Guerrero Magaña, Ulloa Azpeitia & Nesterova, 2016).

In this sense, the vibrating string is a physical system that has a significant role in training students in STEM programs, because it is fundamental vibration analysis of continuous systems and theoretical understanding of complex wave phenomena. Their study also necessary to solve problems in areas such as mechanics or acoustics, when you want to understand the physics that lies behind the sound of stringed musical instruments like a piano, a violin, a cello, guitar, etc. .

This system is also used in industrial instrumentation in fairly accurate and reliable sensors are used to measure various physical parameters such efforts and / or structural deformations in buildings and bridges, including special facilities such as telescopes and nuclear plants. Similarly, in some industrial processes can be production systems in which some mechanism that under certain conditions, can be considered that their behavior is similar to that of a vibrating string is involved.

In particular, an elastic cord tensioned eg, a thin copper wire, which leads alternating electrical current within a nonuniform magnetic field is a nonlinear dynamic system parameters spatially distributed, which allows to study nonlinear effects of complex wave motion and results can transcend various applications. For this reason, this system is of great interest to both experimental and theoretical scientific research.

Historical-conceptual system development

The interest in studying the dynamics of a vibrating string appears since the beginning of Western civilization in ancient Greece, when the philosopher Pythagoras wanted to understand the relationship between the length, tension and diameter of a rope with musical sounds produced when pressed ; he established that these harmonics could be represented by ratios and proportions of natural numbers. His studies were conducted on a stringed instrument constructed for this purpose, the monochord.

But strictly the first mathematical models of a vibrating string approximately twenty centuries later and only academic interest were developed to describe some basic phenomena System (Boyer, 1968); Taylor established in 1715 mathematically physical relations involved in the movement of a vibrating string, concluding that the movement of any arbitrary point in it is equivalent to the motion of a simple pendulum and also earned his period of oscillation; linear wave equations for the study of free oscillations were reported in 1747 by d'Alembert with a purely mathematical approach in 1748 by Euler that more physical intuition, deduced that the initial conditions determined general solution of equation given; system was also studied by Daniel Bernoulli in 1753 and in 1759 Lagrange, both proposed a solution as series.

A century later, in 1859, he based on their experiment Melde reported the presence of parametric excitation; Kirchhoff in 1876 obtained the first non-linear model with an equation integro-differential and analyzed the free transverse oscillations regardless of the longitudinal oscillations. Moreover, Rayleigh in 1883 and 1887 proposed an equation of nonlinear wave to study free oscillations and forced considering the damping(Rayleigh, 1883, 1887).

At the beginning of the last century, in 1910 Raman, Nobel Prize in Physics in 1930, also proposed a nonlinear model to study the forced disappearance of a vibrating string oscillations, also by a stroboscopic study of the movement of the nodes made the first photographic records of standing waves(Raman, 1910).

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During the twentieth century, the issue also prompted the curiosity of some researchers with an interest in understanding the dynamics of certain musical string instruments, highlighting the work of(Narasimha, 1968)He proposed a non-linear model to study a veena; in(Bank & Sujbert, 2005)non-linearities present geometric analyzed on a piano; and Politzer, Nobel Prize in Physics in 2004, published a study of the dynamics of a banjo(Politzer, 2015); just to mention some examples of several recent studies.

Relevance of inclusion in STEM PE

STEM-type courses involve the development of skills and very specific skills that are directly related to the concepts discussed. Specifically talking about mathematics and physics, problem solving is one of the most used to get students to come to the understanding of various phenomena through analysis exercises proposed solution methods. Observe, perform graphic schemes, define initial parameters, select the mathematical model, solve and relate the results with examples and their physical implications encourages students meaningful learning because it requires processes that any taxonomy are considered complex, as they go beyond store or machining solutions, synthesized,

Designing and adjusting educational programs that emphasize the focus on STEM courses, it is consistent with the proposed powers (Tobon Tobon, Pimienta Prieto, & Garcia Fraile, 2010)who point out that it is expected that "the curriculum point to daily and regular practices that promote the integral development of individuals, so that they are able to contribute to solving the various current problems", these problems are concerning various levels such as are the personal, social, organizational, among others. Now curricular courses STEM planning situations requiring strong analytical skills, the difference between traditional education encyclopedic against competency-based approach today has been promoted in the classroom is the ability to contextualize situations study by direct application, thus promoting a systemic and complex knowledge.

For his part in(Biggs, 2006)It noted that learning activities should be adequate, meaning that those whose focus is deep, avoiding superficial methods and practices; it is here that the modeling of a vibrating string provides a complex system whose solution involves several cognitive processes that promote extended abstract, product of a cluster of significant teaching sequences.

Technological Applications

Here are some of the technological applications where a daily basis now and you can see the system of the vibrating string; If the reader wants to know the details of the mathematical modeling of these systems, a review of the primary sources is needed.

Catenary and pantograph system. Around the world there are many cities where you can observe urban electric transport systems. In our country only Guadalajara, Monterrey and Mexico City have this service, and soon will add one more than intercity and will connect Toluca to Valle de Mexico.

Figure 1 Catenary and pantograph system

The power supply for urban electric transport system is via a cable called catenary contact interacting with a device called a pantograph located on top of the train. The dynamics of this mechanical system catenary / pantograph characterized in that the pantograph is displaced variable (or constant) speed along the catenary, acting as an external driving force to transmit the vibrations of the train and causing transverse oscillations on the catenary.

Airlines power transmission. The electrical energy generated in the core of different nature (air, water, solar, thermal, nuclear, etc.) is carried to users through aerial power lines, transmission and distribution.

Figure 2 Overhead transmission lines

In the case of transmission lines, it is stringing cables are of an aluminum alloy and are placed between towers anchor. In some regions of the world, it has been observed in transmission lines the presence of oscillations that have motivated the interest of different interesting to analyze scientific phenomena such as oscillation galloping, among other investigations. The effect is galloping oscillations, horizontal or vertical, high amplitude and low frequency are caused by the action of winds on the constant transmission lines increases during winter when the cable is covered with snow or ice. These oscillations in combination with torsional vibrations are sometimes so great that two cables of different phase may be too close and produce a short circuit, even does occur structural damage installation as wire breakage or fracture of the anchor towers; in any case, maintenance or repair costs rise and power service is interrupted.

Cable Car system. Cable cars are systems to transport people with cabins that move along a cable suspended between two points and are installed in some cities in the world for different purposes.

Figure 1 A cable car system

In our country a cable car system was recently installed in 2016 for use as mass transit in the municipality of Ecatepec Mexico State with an approximate path of 5 km and a height of 35 meters; this system also is used as a tourist attraction in some cities of the Republic. Table 1 shows some characteristics of these systems (data obtained from websites of relevant governments) are as follows:

Year [Installation]	Place [City]	Length [Km]	Height [M]
1979	Zacatecas	0.650	85
1982	Taxco	0.800	175
2010	Copper Canyon	2,800	400
2010	Durango	0.750	82
2016	Puebla	0.688	58

Table 1 Some places cable car system in Mexico

Interest in scientific studies related to a cable car due to the complex wave phenomena that can occur during operation, control and provide security to users. For example, the system installed in Zacatecas has improved this year 2018 by placing new high-strength cables operating at a higher voltage, variable speed and higher user density in the new cabins will increase from two to six.

Based on the above, it is clear that the system consisting of a vibrating string is quite rich conceptual and practical interest for their applications, so a reliable mathematical model is necessary to understand the real behavior of the various phenomena present in a string vibrant.

Modeling of a vibrating string

Because of their diverse areas of application, it is indisputable that the study of oscillations in strings (wires, wires, cables) is of great interest both in acoustics for the design and manufacture of stringed musical instruments, such as the study of dynamic properties of systems running wires elevators, cranes, cable-stayed bridges, trains, etc.(Kumaniecka, 2002; Zhang, Zhu, Lin, & Wu, 2004); also in instrumentation design based on a vibrating wire in order to measure different physical quantities such as stress and strain in structures, viscosity and fluid flow sensors and even scanning particle beams(Arutunian, Mailian, & Wittenburg, 2007, Mei, Lucas, Hole, Lamarque, & Chéron, 2016).

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It is worth mentioning that the different linear and nonlinear models that have been reported are basic approaches or are focused on a particular phenomenon. This is explained by the variety of physical phenomena related to this dynamic system, for example, the hysteresis appears if the frequency scanning is performed forward or backward(Lee, 1957; Tufillaro, Abbott, & Reilly, 1992); Furthermore, if during scanning frequency is forcing very small or moderate amplitude then the linear or nonlinear, respectively resonance occurs; Moreover, the phenomenon of parametric excitation is modulated observed when any of the system parameters(Minorsky, 1951).

The mathematical model of an elastic cord ideal is described by the equation where linear wave is the linear mass density is the transverse displacement, is the time, is the tension in the string; points and apostrophes are in temporal and spatial derivatives, respectively; It is the position along the rope$\mu\ddot{y} = Ty''\mu ytTtzz$(Morse & Ingard, 1968). However, a real thin rope has some properties of a thin bar when the magnitude of the thickness. The magnitude of the thickness and material properties of the string give rise to a variety of nonlinear models that can sometimes result in coupling of transverse, longitudinal and torsional modes when factors such as stiffness, torsion or geometric nonlinearity are taken into account$\varepsilon \rightarrow 0$(Bank & Sujbert, 2005; Watzky, 1992).

Generally, one of the differences between the linear and nonlinear models that have been proposed for the vibrating string is in consideration of a small or large oscillation amplitude. If the amplitude is small, there is little rope stretch and strain can be considered constant during transverse movement. By contrast, large amplitude oscillation can lead to increased stretching and the tension is variable along the entire rope. Moreover, it is noteworthy that in some physical systems can actually arise factors acting in a manner contrary and cause the voltage to decrease, as the case of heating in certain conductive wires of electric current. By including this factor in the mathematical modeling of a vibrating string.

When a wire with alternating electric current is subjected to the action of a magnetic field, the electromagnetic interaction produces the Lorentz force that causes oscillation of the wire. These oscillations are nonlinear; the geometric nonlinearity is caused by stretching of the rope during the oscillations and variation or increased tension in it. Furthermore, the heating wire causes dilation and consequently the decreased tension. These two effects are opposed and act simultaneously.

When considering a perfectly flexible cord, linearly elastic and stretched in equilibrium at a length along the axis; y are the coordinates of transverse movement of the rope; the nonlinear equation of plane polarized transverse wave in a thin string stretched to the length of equilibrium is:$L_0 zxyL_0$

$$\ddot{x} + 2\beta\dot{x} - C_t^2 x'' - C_l^2 x'' \frac{1}{2L_0}\int_0^{L_0} (x')^2 dz = \tilde{f}(z)\cos(\Omega t) \quad (1)$$

Where is the transverse displacement of the string segment at the point in time; It is the angular frequency of the external force which acts in the plane direction; It is the damping coefficient. Transverse wave speed depends on the tension in the rope, while the longitudinal wave velocity depends on the linear elasticity module; It is the mass per unit length of the string in its steady state, is the Young's modulus of the cord and its cross section is the Lorentz force. The term of the integral in equation (1) represents the geometric nonlinearity that comes from the change in string tension due to the variation in length when it oscillates.$x(t,z)zt\Omega\tilde{f}(z)\cos(\Omega t)\, xxz\beta C_t^2 = F/\varrho F C_l^2 = \lambda/\varrho\lambda = YS\varrho YS\tilde{f}(z)\cos(\Omega t)$

The temperature variation in the rope is of the form where is the stationary part of the temperature increase in the rope with respect to the environment, is the amplitude of temperature variation in the rope and the gap between the temperature variations on the rope and the electric current.

And they are the model parameters. After a detailed mechanisms of expansion of the material of the rope and the voltage drop due to the temperature variation affecting the speeds of transverse and longitudinal propagation analysis nonlinear integro-differential equation for the transverse movement it is obtained a thin rope on the plane: $\Delta T(t) = \Delta T_0 + \delta \cos^2(\Omega t - \varphi)\, \Delta T_0 \delta\varphi \Delta T_0 \delta\varphi xz$

$$\ddot{x} + 2\beta\dot{x} - \left\{ \begin{matrix} \check{C}_t^2 - \frac{\lambda\alpha\delta}{\varrho}\cos^2(\Omega t - \varphi) \\ + C_l^2 \frac{1}{2L_0}\int_0^{L_0} (x')^2 dz \end{matrix} \right\} x'' = \tilde{f}(z)\cos(\Omega t) \quad (2)$$

In which it is observed that the temperature variation in the elastic cord (thin wire) linearly manifested through the speed variation of transverse wave; while stretching the rope during its oscillation is expressed nonlinearly through the integral term. Physics equation (2) it includes the interaction of two opposing effects, dilation and geometric nonlinearity.

As a first step in the study model and in order to better understand the role of the warming of the dynamics of the oscillator, the geometric nonlinearity in equation (2) is omitted and the following linear equation of motion of the rope is obtained low harmonic warming synchronized with the driving force:

$$\ddot{x} + 2\beta\dot{x} - \left[\check{C}_t^2 - \frac{\lambda\alpha\delta}{\varrho}\cos^2(\Omega t - \varphi)\right] x'' = \tilde{f}(z)\cos(\Omega t) \quad (3)$$

In a string with fixed ends, the separation of variables leads equation inhomogeneous wave (3) to the equation -th oscillations in normal mode: n

$$\ddot{T}_n + 2\beta\dot{T}_n + \left[\check{C}_t^2 - \frac{\lambda\alpha\delta}{\varrho} \cdot \frac{1 + \cos(2\Omega t - 2\varphi)}{2}\right] k_n^2 T_n = f_n \cos(\Omega t) \quad (4)$$

Equation (4) it is a forced eigenfrequency this modulated parametrically, and damping coefficient regulated oscillator. $\omega_n^2 \beta$

The oscillator (4) is multi-parametric, therefore, and modeling can be enriched even more, depending on your specific configuration and purpose of it. In particular, in the shown model it can be studied in analytical, numerical and laboratory form the forced vibration, direct and parametric resonance and extended to consider the effect of the geometric nonlinearity and movement in both planes.

Conclusions

With the training of professionals in STEM education programs educational institutions contribute to the development of the regions of the country. Consequently, it is the responsibility of teachers to design relevant plans and programs of study, in which it is recommended that the content viewed as a central axis mathematical modeling of real systems that are linked directly to their professional practice. Mathematical modeling as a teaching activity enhances student learning in mathematics and allows you to make sense of each parameter or variable involved in the modeling process.

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