Infinity to the inexhaustibility

Infinito a la inagotabilidad

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Abstract

The notion of infinity is one of the concepts That definitely requires greater cognitive work in mathematics, ITS conception escapes the common logic of the concept of quantity and Its numerical representation, but the transcendence of the concept lies in the key is to Understand the concept of limit. This requirement Makes it essential for the teaching of differential calculus. However, the infinite in daily life is as distant as inert, with no other use as the philosophical and the academic, so it Throughout ITS historical appearance has always Been a controversial and neglected topic. In this article we present an indirect method to infer the basic limits of differential calculus That tend towards infinity, by Means of the logical deduction in a concentration experiment of potassium dichromate Dissolved in water and in an intrinsic way making use of Its property of the inexhaustibility That are able to elucidate the essence Also of the infinite. Dichotomous by nature does not cease to be a pristine concept of humble origins, Which for some HAD to be developed from the need to Exceed the limits of the measurable, while for others it Represents the World That encompasses mathematical knowledge or maybe generated as an answer Disturbing to the insignificance of the human being in the universe

Infinite, inexhaustibility, exhaustive number, immeasurable numbers, differential calculus and infinite

Resumen

La noción del infinito es uno de los conceptos que definitivamente requiere de un mayor trabajo cognitivo dentro de las matemáticas, su concepción, escapa de la lógica común del concepto de cantidad y de su representación numérica, pero la trascendencia del concepto radica en lo fundamental que es para entender el concepto de límite. Este requerimiento lo hace indispensable para la enseñanza del cálculo diferencial. Sin embargo, el infinito en la vida diaria es tan lejano como inerte, sin más uso que el filosófico y el académico por lo que a lo largo de su aparición histórica siempre ha sido un tema controvertido y desdeñado. En este artículo presentamos un método indirecto para inferir los limites básicos de cálculo diferencial que tienden al infinito, mediante la deducción lógica en un experimento de concentración de dicromato de potasio disuelto en agua y de manera intrínseca haciendo uso de su propiedad de la inagotabilidad que sean capaces también de dilucidar la esencia del infinito. Dicotómico por naturaleza no deja de ser un concepto impoluto de humildes orígenes, que para algunos tuvo que ser desarrollado a partir de la necesidad de rebasar los límites de lo medible, mientras que para otros representa el orbe que engloba el saber matemático o tal vez generado como una respuesta ante la inquietante insignificancia del ser humano en el universo.

Infinite, inexhaustibility, exhaustive number, immeasurable numbers, differential calculus and infinite

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Introduction

On any given day the student computing is a concept that breaks everything he knew in mathematics the infinite, all-encompassing and in turn has no value, can be as large as so small, can be conceptualized but in the moment that he represents no longer infinite. Such a concept is not natural in our daily lives and anything not in the ordinary, is related to infinity. Thus we face the student with a concept other novel but that might even seem puzzling motivational for study. However, the student is so confused before his eyes when we try to clarify it giving it qualities as any other number still is not number, adding it developed algebraic operators, multiplying and dividing as if it were, In addition we relate it to zero until after the year 100 d. C. was used in western by Ptolemy (100-170 AD.), That is another number as special features represented only source nowhere vacuum (Babylon, S. III a.) 1 , zero has become common for learners of XX and XXI century, but its nature is generally not well understood.

But back to infinity, this has been addressed by philosophers and mathematicians (eg Fernando Hitt, 2003) .2 mentions the distinction of Kant in the eighteenth century between the infinite potential (buildable) and current (not buildable) infinite, cited quote: That is the infinite potential can be conceived through experience (possibility of going further); however, the actual infinity is constructed by a reflection. Hitt says the same Kantian ideas do not permeate at that time in mathematics which led to a series of misperceptions regarding the convergence of a function; For purposes of our investigation we assume that the practice will help students build their concept of infinity, somehow, our experiment relates the two interpretations which postulates Kant,

Having explained a function that goes to infinity which before the students- believe we made it clear that it is infinite, expanding the explanation we consider it as a numerical entity stressing that is not an ordinary number and without pre-ambulo delve students in differential calculus by defining limit and logically with their operative processes including infinity, which may tend (close) to zero, tend to a constant value or not at the same infinite ?.

This position somehow becomes a substantial problem for the subject of college calculus, it is known that a problem exists in all mathematics (eg Farfan, 2012, Steen, 1988) 3: Although problems mathematics education are many and diverse, the teaching of calculus has been recognized as one of the greatest failures in higher education. The calculation takes a neuralgic place in the latter: its links with both elementary mathematics as advanced mathematics, as well as their role in mathematics and science, do a body of knowledge with theoretical and empirical value essential in education higher.

The boundary of a given function "x" is "and" when "a" tends to "b", thus:

$$y = \lim_{a \to b}(x) \qquad (1)$$

But this "trend" is? It is an expression which indicates how approximate is a value in a function referenciándola with some plan or system, however, when b is inclined to be overly large or extremely small degree of lose track of its value, "b" then tends to infinity. Theoretically, the concept is very simple but its conception is not so, mainly because our idea of number is closely related to the amount, in addition to the intricacy that it used the same concept to the very large or very small at once, what at first glance it seems ambiguous and also confronts our idea of how big and small.

Infinity always has suffered from credibility, Eleatic School (S. VI-V BC) or Megarian (S. IV BC), depending on the context, the question from the viewpoint of truth can not be a product of the assumption under this premise Zeno of Elea sets out the paradoxes seeking to demonstrate their absence and perhaps the best known of these paradoxes is the Achilles and the tortoise where hypothetically "the warrior of light feet", the champion most renowned in the ancient Greek world accepts the challenge of a humble turtle to compete in a race of infinite proportions. Once ready competitors, Aquilesand at the finish line gives an advantage to the turtle for obvious reasons-a stadium to be exact, the turtle goes as fast as her short legs allow, after a long wait.

Aquiles part of the starting line, from then the distance traveled by the warrior will always be sectioned in a middle of the turtle, this provision limits the champion of a thousand battles to go smaller and smaller sections, always half the distance the turtle at the same time, sections to follow are becoming smaller and smaller so the champion of Mirmidones never reach the tortoise. Zenon logically supports his thesis that the tortoise Achilles reach sooner or later, so that the conceptualization of the infinite seems purely speculative.

Zeno's paradoxes related to infinity without doubt call into question the veracity of this concept and each allows us to develop a greater attachment to the concretes of our world, a world in which everyday life rarely extends millimeter beyond, so infinity is outside the everyday context. For Hilbert (1862-1943) 4 existence in mathematics it means free of contradictions and paradoxes seem to contradict reality and infinity. But in defense of infinity we say that all Zeno's paradox uses a measurable dimension and resulting in an uncertain infinite.

In the search for truth and coupled with the concept of denial of infinity, in addition to greater credibility were in the development of science called Greek materialists, who, in their quest to unravel the mysteries of knowledge and knowledge they had to break off all relations with the gods genesis and find as the primary reason for life and the universe to matter. From their perspective the matter was made up as tiny particles that could not be divided over which they called atoms, the appearance of them in the theater of wide knowledge borders seeking knowledge and extensive exponentially the mathematics however intrinsically refuses to infinity when considering the atom however small it will always be ponderable.

Infinity for students is a "Very grandotote, something very lejototes or something very tiny" and between "tototes and Tititos" We appreciate that the concept is quite vague for students: rather ambiguous, to the extent that even at the university level still represents for them object but in all honesty, the fault is not entirely of them, we spent years teaching that every number represents a quantity that every number between itself is one and that all number between zero no solution, while at the show limits basic trying to infinity as a number that does not represent much, but it has many of the qualities of numbers and we can add it to a constant, we can multiply and to divide it also has the quality to add also, to multiply likewise but not be divided between himself,to aggravate the situation by dividing a constant between zero also refer to the boundary tends to infinity, not conforming to the constant and infinite Referring now we engage the where zero, which for practical purposes is equally mysterious infinity.

Javier Perez of the University of Granada, says that for the S. XVIII, The idea of quantity is more general than the idea of number. A line segment, for example, represents an amount, but he is not limited to numbers. The idea of number as an element of a set does not exist in the eighteenth century. For the same reason, a segment can not be separated from its ends and always included. The numbers were interpreted as measures. In Arithmetica universalis (1707) Newton5 writes:

By number we understand not so much a multitude of quantities, such as abstract reason of any amount to another quantity of the same kind we take for unity. An integer is what is measured by the unit, a fraction, that to a submultiple of the unit measures and square root, what is incommensurate with the unit.

But the close correlation between quantity and number survives today what further hinders conception of infinity and the infinite dichotomy in full nature -the mystical and indecipherable continues to elude an inflexible definition.

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Thus, the infinite conceived as incommensurable numbers from Euclid (325-265 BC) into infinitesimal calculus mathematics developed as the rate of change between two variables Newton (1643-1727) and Leibnitz (1646-1716) in the S . XVII and development of the idea of varying amounts of the ratio so far numbers a physical reality that can only be interpreted as an amount which escapes the concept amount to lose proportion of the measurable and become such an ethereal notion as intangible, leaving us as the only measure for the Archimedean property abstraction of numbers exhaustive Eudoxus of Cnidus (400-347 BC) the inexhaustibility, ie lead to a constant up it disappear almost entirely, on the understanding that although we can not see it is still there, showing that however small the constant is not zero, but if you tend to him.

Infinity plays an important role in the academic context of mathematics (eg Sacristan 2003) 6 Infinity is in a variety of contexts and areas of mathematics: from geometry to set theory, to the concepts of sequences, series and limits. But the context and situation affect the interpretation you have of infinity, so thereal purpose of the experiment is the introjection of the infinite concept, necessary for the issue of limits on the subject of calculation

Differential Race Nanotechnology Technological University of Ciudad Juárez (UTCJ), for which we use an indirect method in the laboratory of chemistry at the same university, practice although very simple was designed so that through ownership of the inexhaustibility approach the definition of infinity, Courant-Robbins, 1941) cite the demonstration of Cantor (1845-1918) for inexhaustibility,

Given any set A is possible to construct another set B with a higher cardinal numberAlthough the theorem is very general Cantor shows that there will always be more, or less depending on the context- to another. Hitt7 Fernando (2003), however, not enough to know the operating process for the limit of a function, which argues: To learn the concept of limit it is not enough to master algorithmic processes associated with it:

It also requires passing through obstacles that do not allow us to distinguish the concept of infinite potential from the current infinity in its coherent use in the mathematical activities of infinite processes. In our case, these obstacles must be overcome through practice and observation. The design of strategies to make knowledge a transcendent knowledge are many and very varied, (eg Sacristán, 2003) 8: So we must take into account that there are approximations to infinity through visual-geometric models or through processes that can be described in purely symbolic-algebraic terms. We opted for the theoretical-practical-visual method in the laboratory, after the theoretical introduction in the classroom. The concept of infinity is fundamental in mathematics, particularly in areas such as calculation where infinite processes are part of the concept of limit. (Sacristán 2003) 9. He adds: From the point of view of mathematical theory, many of the difficulties presented by the infinite have been resolved as the concept has been defined as a mathematical object with its own operating field, particularly from the works of Bolzano (1851) 10.

In the practice of concentration designed to approach the concept of infinity, we maintain a one-to-one correspondence between the solvent and the solute, which becomes a restriction for the objective of the practice because both are finite quantities, however, from the beginning it was clear that even with "n" tests we would obtain to infinity, the approach to the concept would lie in the fact of the exhaustive theoretical amount of evidence that would represent the extremely small presence of K2Cr2O7 and the intuition that for so many more that we did, it would follow I presented.

In the construction of the concept of the infinite we bet on visualization and above all on intuition, we risk the challenge of each student being able to shine beyond the dividing boundaries between the strictly physical and the purely mathematical.

Indirect method

In order to define the concentration the amount of solute found in the mixture is measured, for which 1gr of K2Cr2O7 (Potassium Dichromate) is poured into 100 milliliters of water and the concentration is calculated, the new mixture is taken 10 milliliters, to which 90 milliliters of water are added to take the mixture on each occasion to a volume of 100 milliliters. K2Cr2O7 is a salt of a very striking orange color, it is soluble in water which allows to visually compare the amount of solute dissolved in the solvent, for practice this part of the experiment is very important because it reinforces the visual factor of the loss of solute In the concentration, in addition, the construction of a table of comparative values reinforces the concept seen in the basic limits, in his schematic book by Joan Costa11 (Paidós, 1998) external that: The diagrams are seen as messages: a "significant everything" in a single moment of perception, interpreting as diagrams any graphic element that reinforces the conceptualization.

Figure 1 Nanotechnology students physically checking the decrease of solute in the solvent

The practice is reinforced with the construction of a table of values of both the solute, the solvent and the concentration, the underlying objective is the introjection of the concepts that relate to infinity with the limit:

$$f(x) = \frac{K}{0} = \infty$$

$$f(x) = \frac{K}{\infty} = 0 \quad (2)$$

In the photograph of Figure. 1 the students of the nanotechnology career of the UTCJ, physically testing the decrease of K2Cr2O7 in an increasing volume of water.

However, the practice is developed around a physical property of potassium salt and each of the participating students take pains to measure exactly the amount of mixture they are going to gauge, ensuring that there is no lack of any of the components of the process and comparing at each step the coloration of the mixture that determines the concentration in the combination of compounds.

	Solute gr	**Solvent** Ml	**Concentration** Gr/ml
1	1	100	1(10)-2
2	.1	100	1(10)-4
3	.01	100	1(10)-6
4	.001	100	1(10)-8
5	.000,1	100	1(10)-10
6	.000,01	100	1(10)-12
7	.000,001	100	1(10)-14
8	.000,000,1	100	1(10)-16
9	.000,000,01	100	1(10)-18
10	.000,000,001	100	1(10)-20
11	.000,000,000,1	100	1(10)-22
12	.000,000,000,01	100	1(10)-24
13	.000,000,000,001	100	1(10)-26
14	.000,000,000,000,1	100	1(10)-28
15	.000,000,000,000,01	100	1(10)-30

Table 1 Table of the amounts of solute, solvent and concentration of the mixture K2Cr2O7 and water

Similarly the construction of the table would be sufficient to demonstrate the amount of solute that remains in the water with each capacity without using a salt so striking allows them to guess the amount of salt remaining by coloring which is going elucidating to the extent that after the 10th test, the mixture is completely crystalline, which in some way leads them to conclude that the amount of water tends to infinity while the amount of K2Cr2O7 remains constant and is lost in both solvent that tends to zero by function:

$$f(x) = \frac{K}{\infty} = 0 \quad (3)$$

On the other hand, the concentration of the mixture in the experiment represents the function:, where the amount of K2Cr2O7 tends to zero while the solvent represents the constant and the concentration tends to be infinitely small. A bridge has been laid between the algebraic construction of the concept and the physical construction of praxis and the question arises: will the same thing happen to the student's cognitive level? $f(x) = \frac{K}{0} = \infty$

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Thanks to Euler (1707-1783) that in the eighteenth century the calculation of geometric quantities and equations in calculating functions, teaching has also been gradually reformed and current students identify that any process can be transformed into a function. (e.g. Suárez 2008)12

The photograph below shows 12 of the 15 phases of capacity to determine the concentration of potassium dichromate dissolved in water, the coloring of the mixture indicates the amount of solute that have in each phase and the result of carrying a huge amount of solvent the mixture logically only to demonstrate the difference between the two, with the understanding that it is by no means infinite, without change, the observation color transformation mix visually shows the pupils as the concentration the solute decreases the degree of not showing their presence from the test 11 the following tests after the eleventh-some twenty students have brought him aforos-showed no change in coloration. For the project, the observation is critical, (eg Puente, 2004) 13,*Reflective observation is to open our eyes to our senses perceive reality around us and question us through reflection considerations that this observation in the form of ideas, objects, goals, experiences, content or behavior really means. Observe assumed an attitude of attention, search, learn the reality*. We believe that the degree of abstraction by visual change of color has a high impact as reflective observation, which take students to build a solid infinite concept, knowing that in schools, theoretical developments on the issue are not common conception and intuition let each student. (Sacristan, 2003, Moreno and Waldegg, 1991) 14

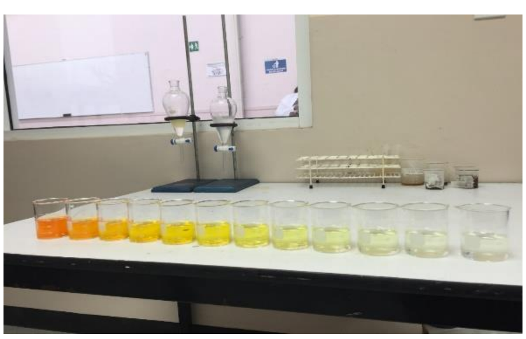

Figure 2 Twelve samples visual phenomenon of concentration of a potassium salt in water

For reporting the students career UTCJ

Nanotechnology practice, they had to answer questions like:

-The color indicates the concentration of dichromate K2Cr2O7 dissolved in water?

- The clear water indicates that there is no presence of K2Cr2O7 in the mix?

-It significantly increased the amount of water passing the K2Cr2O7?

-It significantly increased the amount of water passing with concentration?

-Development to mathematical function in the process of gauging of K2Cr2O7 and water?

- Of the determination of basic limits seen in class to practice in the laboratory. As relates to the functions:

$$Y\ ?f(x) = \frac{K}{0} = \infty f(x) = \frac{K}{\infty} = 0 \qquad (4)$$

What is infinity?

Results

Practice the following comments related to the inexhaustibility and infinite were obtained:

- Did the increase in water significantly increase the K2Cr2O7?

100% of students responded that the concentration decreases.

11% dichromate tends to zero.

- To the increase in the water increase concentration (defined as the ratio between solute and solvent)?

78% answered that the concentration decreases.

11% the concentration approaches zero.

33% will always be K2Cr2O7 in the mixture.

Another 11% said that the concentration of K2Cr2O7 always be constant, referring specifically to the amount of solute mass does not change in the experiment.

And finally they were asked:

What is infinity?

89% responded that it is a number.

6% which has no limit

6% which has no value.

6% which does not have much.

11% inexhaustibility.

Conclusions

Go the theoretical concept of infinity in the classroom to an experimental laboratory practice pending intuition to do its part, it is a risky yet impressive quality of answers given by the pupils to infinity, as evidenced the percentages obtained with respect to their ownership of inexhaustibility and over the same infinity. Generally concluded that infinity is a number within its properties does not represent any amount that divided itself is not one which is bivalent both large and small yet is univocal having only one meaning and if there is, at least mathematically.

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