

A look at the learning and teaching of geometry by Van Hiele and Brousseau

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Abstract

The research "A look at the learning and teaching of geometry, by the hand of Van Hiele and Brousseau" is a partial result made for a final document titling for undergraduate in elementary school. The professional practice in which the action plan and strategies it implemented was implemented at Manuel Rangel Martínez Elementary School in Loreto, Zacatecas. This report generates a reflexive analysis on the teaching and learning of geometry from the implementation of strategies and instruments designed based on the planning devices of Van Hiele model and Brousseau Theory. The results highlight which of the devices may have more strengths in acquiring new levels of learning in students. At the end of the analysis, it was concluded that there are more strengths to work geometry with the Van Hiele model.

Theory of Didactic Situations, Van Hiele Model, Teaching Geometry

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Introduction

The present research work was carried out during the internship period of the last semester of training for graduates in primary education. It was developed with a sixth grade group from the elementary school "Manuel Rangel Martínez" located in Loreto, Zacatecas.

The approach of the problem aims to make a justification of why the choice of research topic, as well as the way in which the plan and study programs are linked to the theme. Here is also a small investigation about the authors who contribute to the subject.

Also, there is an analysis of the results and conclusions reached by some of the most important authors that are related to the subject of study (geometry) which for this study can be considered as more important: Theory of Didactic Situations by Guy Brousseau and the Van Hiele Model. From the above, not only are the theories reviewed as presented, but an analysis is made of how these theories can be applied in classrooms.

The methodology that was used was the action research. The above gives a guideline to generate an intervention plan with which we can attack the problem by making a reflective critical analysis of professional practice. Based on the above, the activities to carry out the research are presented.

Finally, a collection of the results obtained from the implementation of the action plan is made and an analysis is made to identify the achievements, weaknesses and strengths. From the analysis, the main conclusions reached are presented.

Problem

Justification

Mathematics today are very important for the development of all people in any field of study or work or even in everyday life. In our daily life we are always surrounded by objects, shapes and designs in which geometry is involved. Since we are children, we interact with these forms, not only in our environment, but also with toys or artifacts that have some geometric properties. It starts from this relationship that exists considering that the teaching of mathematics begins with the observable and what we observe in the environment where the different forms and designs in which geometry is immersed are manifested.

In the particularity of the study of space geometry today as is traditionally done, we can say that the relationships between the different dimensions are studied: Dimension 1 that includes curved lines and lengths. Dimension 2 the surfaces, areas etc., Dimension 3 three-dimensional objects, solid bodies, volumes; even different or more complicated models that are used scientifically that are only used in higher education. (Alsina Catalá, Bourgeois Flamarich, & Fortuny Aymemmi, 1997)

Mathematics needs new forms of teaching so it requires more specialization in its didactic, which can not only be based as traditionally done on the blackboard and chalk. And now more than ever with the new contributions of the theories of mathematics teaching, as well as the new technological instruments that facilitate the learning and understanding of mathematics.

We know that geometry in most schools in our country, teachers rarely, both in training and in service, show emphasis to this branch of mathematics because it is believed that it is not important as the learning of arithmetic (addition subtraction multiplication division) or some other content. Considering the above, the immersion of the learning of this important branch of mathematics is crucial. When carrying out an in-depth analysis on the teaching and learning of the characteristics and properties of the figures and geometric bodies in which they must be taken into account according to: (SEP, 2011) The axes of training as:

- Identification of parallel, secant and perpendicular lines in the plane, as well as of right, acute and obtuse angles.
- Location and stroke of the heights in different triangles.
- Construction of geometric bodies with different materials (including cone, cylinder and sphere). Analysis of its characteristics regarding the shape and number of faces, vertices and edges.
- Distinction between circle and circumference; its definition and various forms of stroke. Identification of some important elements such as radius, diameter and center.

And although our plan and study programs, like the textbooks of the students, have contents in which the previous axes of training are worked on, we need to improve our teaching strategies.

In the emphasis on the didactics of geometry, we must understand that for the student the first element in this matter is that of visualization since geometry relies primarily on images, drawings and different geometric shapes that exist around it, taking into account that not only by visualization can we learn geometry.

Starting from this idea it is important that the teacher generates strategies in which the student is able to identify the immense geometry for which we are surrounded, understanding in a general way the characteristics of the figures.

Subsequently the student strengthens their learning by communicating geometric information, for example written, graphic or orally. For this students must appropriate the concepts to communicate the visualizations they have made for example: the window is a square and the door a rectangle. In this part the students must justify what they know is a square, in this part is where students begin to know the characteristics of geometric figures.

In the knowledge of geometric space we must distinguish two modes of understanding and expression, which is done directly, which corresponds to geometric intuition, of a visual nature and that is carried out in a reflexive, ie logical, nature verbal. These modes of knowledge, although very different, are complementary. The first is creative and subjective, while the second is analytical and objective. (Alsina Catalá, Burgués Flamarich , & Fortuny Aymemmi, 1997, pág. 15)

Another aspect in which we see the importance of researching this topic is the proficiency of teachers' content. This problem is observed when teachers limit the geometry mainly to topics such as: perimeter, area, volume, some others we consider that with the fact of learning the shapes, the figures, their name and definition, we have a complete learning of the geometry, reducing this important branch of mathematics only to figures and definitions, which makes it monotonous and only of memorization that does not really lead to a true analysis of the concepts and contents worked on. As mentioned in (SEP, 2011) possessing only knowledge or skills does not mean being competent.

The main aspect to investigate is the planning of the geometric contents that are used in schools. For this we remember that there is the Theory of didactic situations that is one of the best devices in terms of the planning of the subject of mathematics which includes the geometry. In contrast there is the Van Hiele model, which his study refers to the learning levels of children in geometry specifically.

When mentioning these two interesting planning devices, you can carry out an investigation by contrasting them to analyze the results that each of them throws, evaluating the difficulties and achievements that we can obtain according to the learning that the students want to achieve, In turn, in the Van Hiele device, the level of reasoning that is intended to be achieved in a group of students. When performing this analysis, assess which device is best suited to obtain the learning in the discipline of geometry in primary school.

Formulation.

The research question that guided the research work is:

How do sixth grade students appropriate geometrical concepts emphasizing the classification, characteristics and properties of triangles considering the planning devices of the Van Hiele model and the TES (Theory of Educational Situations) by Guy Brousseau?

Diagnosis.

The research was developed with a group of 6th grade of primary education that has a total of 29 students. To generate the diagnosis was taken into account the levels of reasoning proposed by Van Hiele:

	Explicit elements	Implicit elements
Level 0	Figures and objects	Parts and properties of figures and objects
Level 1	Parts and properties of figures and objects	Implications between properties of figures and objects
Level 2	Implications between properties of figures and objects	Formal deduction of theorems
Level 3	Formal deduction of theorems	Relationship between theorems (axiomatic systems)

Table 1 Levels of geometric reasoning by Van Hiele

To obtain the diagnosis on the knowledge of the basic notions of the geometry considering the Reasoning Racks of Van Hiele. Activities were used in relation to the characteristics and properties of the triangles. The results showed that most of the students are in Level 1 of reasoning.

Action hypothesis

The planning devices (TES and Van Hiele) can be used to achieve the expected learning of children. But in Van Hiele's model there will be a better understanding of the geometrical contents in the students.

Objectives

1. Carry out an intervention plan, in which you can assess the importance of planning for what two authors are going to take into account: First, the Van Hiele model as a planning device to contrast with the Theory of Educational Situations of Brousseau.

2. Understand which are the three essential aspects on which geometric education is based and analyze how these aspects can be developed in relation to learning and understanding that discipline.

Theoretical framework

In the subject of the teaching of mathematics one of the most important authors considered until our present time is Guy Brousseau with the Theory of Didactic Situations. This theory of teaching seeks to create the necessary conditions generated by the teacher, so that the student acquires mathematical knowledge. With the understanding that students do not acquire this knowledge spontaneously. There are authors with a more exhaustive analysis on the theory, for the interests of the present work, it is retaken what is considered more pertinent to the investigative interests.

This theory is based on Piaget's constructivist theory. According to Brousseau (1986) cited by (Panizza, 2011)

The student learns by adapting to a medium that is a factor of contradictions, of difficulties, of imbalances, a bit like human society does. This knowledge, the result of student adaptation, is manifested by new answers that are proof of learning.” (pág. 3)

This theory gives vital importance to the role played by the situation in the construction of mathematical knowledge as described below Brousseau (1999) cited by (Panizza, 2011):

We have called the situation a model of interaction of a subject with a certain means that determines a given knowledge as the resource available to the subject to achieve or preserve in this environment a favorable state.

Some of these "situations" require the "prior" acquisition of all the knowledge and necessary schemes, but there are others that offer a possibility to the subject to build for himself a new knowledge in a "genetic" process. (pág. 3)

From this the didactic situation is derived which is a situation that favors the teacher so that the students construct a certain mathematical knowledge Brousseau (1982) cited by (Panizza, 2011) defines it as:

A set of relationships explicitly and / or explicitly established between a student or a group of students, a certain medium (which eventually includes instruments or objects) and an educational system (represented by the teacher) in order to get these students to appropriate of a knowledge constituted or in process of constitution. (pág. 4)

The main objective is that teachers design situations in which they give the opportunity to the student to build their own knowledge. For this, the teacher, rather than a problem solver, becomes a "returning teacher" (Lizarde, Hernández & Zúñiga 2017), which means that he manages the conditions and the learning environment, but at the same time allows the student to maintain the responsibility and interest in their learning, there are times when the student will be alone with the resolution of a situation, must leave the student to find the corresponding solutions, thereby generating conflict, even if they are not correct and not limited to a form of solution.

Another phase of the process is the a-didactic situation in this part is the student's learning since students must find the relationships that exist between the procedures used and the results obtained.

The situation that the teacher decides to implement must lead the student to solve it by acting, speaking reflecting from the moment he appropriates the problem. According to Brousseau (1986) cited by (Panizza, 2011).

The a-didactic situation term designates any situation that, on the one hand can not be conveniently mastered without putting into practice the knowledge that is intended and that, on the other hand, sanctions the decisions the student makes (good or bad) without teacher intervention in what concerns the knowledge that is put into play.” (pág. 4)

The return is the part where the teacher acts when a question arises of the student facing the situation, not with the silence to the question, should encourage the student to solve it but not giving the answer, can mention to the student that there are different ways to resolve the situation and then you can check your results.

Another notion that is of vital importance in this theory is the didactic variable, these are the conditions that the professor gives for the resolution of a situation, these conditions can be varied at will of the teacher to modify the strategies of resolution of the problem.

(...)didactic situations are theoretical objects whose purpose is to study the set of conditions and relations of a well-determined knowledge. Some of these conditions can be varied at the will of the teacher, and they constitute a didactic variable when, according to the values they take, they modify the resolution strategies and consequently the knowledge necessary to solve the situation (Panizza, 2011)

With respect to the Van Hiele model:

Known as "the Levels of Van Hiele" began to be proposed in 1959 and has been the subject of abundant experimentation and research that have led to introduce various qualifications, but still continues to be useful to organize the geometry curriculum in primary and secondary education. (Godino & Ruiz, 2002)

Another of the most important authors in the teaching of mathematics emphasizing in the discipline of geometry is the Van Hiele model designed by Dina and Pierre Van Hiele although at the premature death of Dina, her husband was in charge of disseminating the model in the year of 1957. The main idea of the model that he mentions us (Fouz & Berritzegune de, 2013) "the learning of Geometry is done through certain Leveles of thought and knowledge", "that are not associated with age" and "that only one Level can be passed to the next" this model can be divided into two broad aspects:

Descriptive: in this Level the author tries to explain how students reason about mathematical contents. This is done through the definition of five Levels of reasoning.

The Levels constitute the fundamental contribution of the model. It is established that the way in which geometric (mathematical) concepts are conceived is not always the same and varies when progress is made in the understanding of geometry (of mathematics). (Gutiérrez & Jaime, 1998, page 27)

Bearing in mind that Level 5 is considered an unattainable level for students, many of the researches indicate that students who have not yet entered a university can reach the first three reasoning levels. It is important to consider that a student can be in a Level or another different depending on the content that is being worked.

Methodology**Investigation action**

The Research-Action paradigm considering (Latorre, 2003) is relevant to enhance research that enhances the improvement and understanding of the practice of teachers. This methodology is characterized by investigating the design of pedagogical initiatives that have the intention of improving the educational practice, so that they undergo rigorous evaluations in the sense of constantly modifying them for their implementation. The evaluation of the strategies implies the use of instruments that help to observe and improve them, so that the researcher must conceive an introspective vision about his work in a self-reflective and self-critical sense, this allows the professional to advance in the research process, because is able to see at what moments the action needs a regulation or adaptation for its timely development.

Its cyclic process allows the observation, the execution of the actions and the reflection of them. That is to say that when investigating the practices, actions are planned, however the research in them allows self-reflection and this leads to the redesign of strategies, which are subjected to evaluation again, generating a loop that leads to the constant restructuring of activities for to intervene. This methodology is flexible in the sense that it allows you to understand and plan, at the same time, the practice of the teaching staff. It not only allows you to diagnose, it also leads you to how to intervene in certain contexts. According to Elliott (1993) cited by (Latorre, 2003) affirms that the role of the researcher in Action Research is self-reflecting on the quality of their teaching, that is to analyze in a critical way their actions in order to value them and determine them again if it is necessary.

In conclusion, the methodology allows to know and symmetrically change the educational practice. The investigation is oriented to investigate primarily the qualitative aspects, since a diagnosis of the social context preferably requires data that show conditions that deal more with the language than with the numbers Pring (2000) cited by (Latorre, 2003). In synthesis, the methodological process consists of first, identifying the initial idea to work (Diagnose), that is to detect the problematic that is desired; The next thing is to recognize the implications and resources that are available to carry out the analysis; the general plan is designed based on the needs that the situation requires, this is structured in steps of action considering significant activities that favor the good development of the project; the strategies of the first step are implemented, the execution is reviewed, its effects; When reviewing, the weaknesses and strengths of the first step are recognized; it is redesigned and at this moment a cycle is finished; once again the corrected plan is analyzed, the steps of the general plan are taken to action, which are evaluated when they are completed, with the intention of recognizing and explaining the failures in the implementation and its effects; the analysis is contrasted with the main idea, in the sense of checking that the project does not go beyond the general intention; the initiatives are re-planned and they are applied, reviewed to reconcile the strengths and weaknesses, generating another cycle. Elliott (1993), citado por (Latorre, 2003)

Instrumentation

The activities that were designed were divided into one per month. So that they did not intervene in the planning and perform them individually with the whole group. All these activities were closely related to the learning and teaching of geometry.

In the elaboration of the last two plans, an attempt was made to solve the lack of learning that was identified in the students according to the diagnosis and activities carried out. These activities were planned based on: the first in the phases of the Van Hiele and the second with the Theory of the Educational Situations of Guy Brousseau. They have the main objective of leading the student to build the learning of the classification of triangles taking into account the size of their sides.

In the application of these activities the division of the group into two parts was considered, the group has twenty-eight students, to select the first ones was done in such a way that not only the students with the highest academic performance remained in the same group.

Results

Previous knowledge

Whenever students build a new meaning they are learning new knowledge. The student to solve a situation must use their previous knowledge, which at the end of the situation this knowledge will restructure, modify, expand or delete. This knowledge may vary according to many characteristics, for example; each child can have different conceptions that are influenced according to context, family, knowledge of previous degrees among others. At the same time, the activation of this knowledge allows the student to start detonating interest in the new topic in which they will be ventured.

According to the above (Díaz & Hernández, 2002) they state that:

Explicitly indicate to the students the educational intentions or objectives, helps them to develop adequate expectations about the session that is covered, that is, before initiating the activity the intentions of the class must be explained, or, some activities that open guidelines to the content that will be developed, for this case the understanding and resolution of additive problems, that being five didactic situations, it is necessary to design one for each case and according to each didactic variable that is used in the problems. In this regard, and according to the aforementioned on the importance of prior knowledge, we proceed to analyze the relevance of each activity that was developed as a strategy to identify and relate the knowledge of students with new knowledge. To begin to know the previous knowledge of the students began to ask about the characteristics and general properties of a triangle. Through the following questions: What is a triangle? How many sides does it have? Are their sides the same? What types of triangles do you know? Some children answered very close and accurate answers to the questions, while some others had no idea about the subject. The questions that were asked served to know the knowledge that the student had about the general characteristics of the triangles, these questions were focused in relation to the main objective to know if any student already knew the classification of the triangles. Below is a section cut of the record that is given at the time of knowing the prior knowledge of students based on the planning of the Theory of Didactic Situations.

Mo. Does anyone know what a triangle is? Raising your hand to participate.

Demian. It is a three-sided figure with different scalene isosceles and equilateral shapes.

Uriel. It is a relative of the geometric body and the name of the triangle is a geometric figure.

Carlos. It is a three-part figure that can be made in different ways such as pyramid

Mo. How many sides does the triangle have?

Aos. Three

Mo. Are all its sides the same?

Aos. No

Mo. Why not?

Uriel. Because there are different types of triangles and each triangle serves one thing

Demian. No, because there are different types of scalene and equilateral isosceles triangles. (Registry 1).

We must recognize the importance of prior knowledge in the development of mathematical activities: on the other hand, in the constructivist approach the use and destruction of previous knowledge are part of the act of learning; use is spoken when the ideas that students already possess can be used to solve the problems that are presented to them, since in the action phase the students put their implicit knowledge into play, likewise when it is spoken of destruction it refers to those resources that the students have learned badly about the comprehension of problems and that therefore will not help them to solve them, in both cases there is an act of learning, that is, it is learned from and also against what they already know, so "new knowledge can only be made by modifying precedents" (Chamorro, 2005, pág. 23).

Previous knowledge in the phases of Van Hiele

In this model to try to know the previous knowledge of the students, first it was mentioned to general traits that geometry would be worked after the following questions were asked: Where is the geometry used? What geometric figures do you know? Do you know what a triangle is? In a triangle all its sides are equal? How many types of triangle are there? The majority of the students related geometry to figures. When asking about what figures the students knew, the first answer they gave was the triangle. From there it was split for the next question that if they knew it was a triangle, a student mentioned a figure with three sides and three angles. All these questions aroused in most of the students the interest of what would be worked on in this subject.

In trying to explain what happened in the rescue of previous knowledge, this model mentions that it tries to demonstrate to the student the presentation of the topic that is going to work in general asking about the knowledge of the macro to the micro so that the students go coming into contact with the content to teach that in this case is done by the last question about the classification of triangles. On the other hand, it is about finding out what students know about the topic that will be addressed, for this questions are asked about the subject we realize that all relate geometry with figures.

In the learning phases of Van Hiele, students are introduced from a broader perspective of knowledge through group questions in which the content to be addressed begins to be addressed from the geometry to continue with the type of figure that you want to work in this case the triangle, from there it is identified what the students know about its characteristics and at the end it mentions the main content that one wants to learn.

Below is a section cut of the rescue of prior knowledge in the information phase of the Van Hiele model.

Mo. Today we're going to work with geometry. Does anyone know what geometry is?

Aos. Yes.

Joshua It's about the angles.

Gabriel. It is a number system that helps us measure figures.

Kevin. They are figures.

Mo. Where is the geometry used.

Aos. In the figures

Joshua If for example in the door with the angles.

Mo. What geometrical figures do you know?

Kevin. Triangle, square, trapezoid, circle and cylinder.

Mo. They know a figure that has only three sides

Aos. The triangle

Mo. Do you know what a triangle is?

Gabriel. A geometric figure that has three sides and three angles.

Carlos. A geometric figure that has three sides and three angles such as isosceles.

Mo. In a triangle all its sides are equal?

Kevin. Not because there are three types of triangle, the normal that has three equal sides the scalene that no side is equal, the isosceles that are two equal sides and the other is not

Mo. How many types of triangle are there?

Kevin. Three the normal scalene and isosceles. (Registry 2)

In the "questions / information" phase according to (Fouz & Berritzegune de, 2013), it is about determining, or approaching as much as possible, the real situation of the students. This phase is oral and through the right questions it is about determining the starting point of the students and the way forward of the following activities.

Phase of action. Autonomous work of the child

In this phase is where the child begins to interact with the problematic situation and the materials that will be used, for this, it requires the staging of the implicit knowledge with which the student counts. The child begins to enter the field of study through the materials provided which are the straws of different colors that were distributed to each of them. In this phase is where the slogan is presented, it is one of the main aspects to be taken into account in the development of an activity since this is what defines the course the class will take, it should be as clear as possible so that the can understand all the students and so the activity can be carried out as previously planned. In order to achieve this phase in the activity, 20 straws of 4 different sizes, 20, 15, 10 and 5 centimeters long, were distributed to each student. Each group of straws of the same size should be of a different color. Each student began to explore the straws. Being all organized with the straws the slogan was given so that they could start working.

Action phase in the Van Hiele model

This phase in the Van Hiele model is known as the Directed Guidance phase, as this phase mentions, students begin to explore the field of study through the material that has been provided to them. To start the action in this planning the students were given the slogan about what is going to be done, to check if it is understood, two children are asked to explain what we are going to do. At this moment the students begin to order the straws by colors others by sizes some others stick the straws with each other and make figures.

This phase begins to put into play the previous knowledge of the students in relation to geometry as they begin to interact with the materials and from there they begin to consider the strategies they can use to solve the situation based on previous knowledge who already have.

Directed orientation. This is where the importance of the didactic capacity of the teacher is most needed. From their experience point out that the performance of students (optimal results versus time spent) is not good if there are no specific, well-sequenced activities for students to discover, understand, assimilate, apply, etc. . the ideas, concepts, properties, relationships, etc. that will be the reason for their learning in that Level. (Fouz & Berritzegune de, 2013)

Validation in the Van Hiele model

The validation phase or as in the Van Hiele model can be adapted to the explicitation phase, at this moment it is when two or more students. They present the proposals or affirmations to which they arrived in the process of the action phase, these are submitted to the assessment of the group so that they have the capacity to accept or reject them or ask for explanations about what could not be clear to them.

To generate this phase in the students, it must be taken into account that they already have the results of the resolution of the situation. When having the triangles elaborated the students are asked to go to the pintarrón to capture the triangles where they should go according to the classification considering the slogan that was given to them at the beginning. For which in the action phase by the teacher was placed a board that was divided into 3 parts. Each child will have to order the triangles in the Table according to their characteristics.

At this moment the three children pass to classify each of their three triangles where he believes they correspond and they are asked why they were accommodated in that place. Group members are given the opportunity to ask if they have any doubts as to why they accommodated him in the place he believed. At the end of the arrangement of each of the triangles by their partners they are asked for each of the triangles that are on the board if they agree on the arrangement that was made.

When counting on the resolutions of the students, they should choose to go to the painting to explain the procedures used for its resolution, as teachers we must consider which students, have reached a resolution in a less successful way to gradually reach the most close to the solution of the problem, considering that the students are comparing the results of their peers. To assess the decision of their colleagues to put their triangles in a certain place of the Table, the group was given the opportunity to ask about the doubts they had about each of the triangles, as well as if they did not agree to argue about their refusal to accommodate, In this way, we can see what is correct or what is wrong by valuing the participations of each of their classmates and comparing their results guided by the teacher.

It is crucial that the teacher to go through the seats in the action phase or free orientation, consider who has reached the most successful procedure for resolving the problem, and from that moment go to decide who will happen, considering that these students are in some wrong results to the most successful, in this way they are gradually seeing reflected the mistakes made by each of them to generate meaningful learning in which students themselves realize what they are wrong or correct.

Explanation (explicit) It is a phase of interaction (exchange of ideas and experiences) between students and in which the role of the teacher is reduced in terms of new content and, nevertheless, its action is aimed at correcting the language of the students according to the requirements of that Level. Interaction between students is important since it forces them to organize their ideas, analyze them and express them in a comprehensible way for others. (Díaz & Hernández, 2002)

Validation based on the TES

In the theory of didactic situations, validation is one of the primary phases in the construction of knowledge. For this in the instrumentation activity this phase was carried out in the following way. First, each child was asked to select one of their triangles. Previously, a table had already been placed in which they would order their triangles according to their classification 1: Triangles of two identical strawberries and one different. 2: Triangles of three unequal straw 3 Triangle with equal straw. After completing the ordering of the triangles in the place designated by the student, several were asked about why he determined that he should go there and that he should argue his answer.

After this the students were questioned that if any of them believed that a triangle was set in a bad place, they were given the opportunity to go to the blackboard to rearrange according to what they thought was their accommodation, clearly arguing their answer of why and if their partners were in agreement confronting everyone's procedures.

It is important that the children select what are their best strategies or what they think are the right ones for the demonstration to their classmates and thus be able to give arguments of them. The comparison of the results between the groups helps us to validate the results, in this way the students are discovering about the mistakes that were made in the classification of the triangles. For this, all together as a whole build knowledge but individually they realize how they failed in their classifications.

Validation situations: two students (or groups of students) must state assertions and agree on the truth or falsity of the same. The statements proposed by each group are submitted to the consideration of the other group, which should have the capacity to "Sanction", that is, be able to accept, reject, ask for evidence, oppose other assertions (Panizza, 2011)

Institutionalization phase in the Van Hiele model

This phase is consistent with the integration phase marked in the Van Hiele model. In order to carry out this part of the planning, the students were explained that we would consider each of the straws that are part of the triangles as if they were the sides of the triangles.

After this it was explained to them that there are three types of triangles according to the classification of their sides which are: the one that has all its sides equal that is the equilateral triangle, the one that has all its unequal sides that is the scalene and the one that has two equal sides and not unequal which is the isosceles. They were asked if there was any doubt about the classification of the triangles for what the students said they had already understood how each of the triangles was.

The students' explanation of the three types of triangles was relevant since they had identified them but only by measuring their sides they did not have the concept of each of them and thus they could appropriately take the name of each one. It was found that students could have consolidated the concept of the three types of triangle by a last activity in which they were asked to draw on a sheet of the machine the triangle they wanted and put their name as shown below:

The first important idea is that, in this phase, no new content is being worked on but only those already worked are synthesized. It is about creating an internal network of learned or improved knowledge that replaces the one that already possessed. As a final idea we can point out how in this structure of activities can be integrated perfectly recovery activities for students / as they present some delay in the acquisition of geometric knowledge and, on the other hand, adequately remaking the groups deepen something more with those students The best performance Although the evaluation activities have not been made explicit, they would also be easily integrated into this structure of activities (Díaz & Hernández, 2002).

To bring to the students' understanding of what was worked on, relating the activities carried out. They were given the concept of each of the triangles according to the classification of their sides according to what they built. The students were told to consider each of the straws from their triangles as the side of a triangle. And they were explained that the triangle with 3 equal sides was called equilateral, that of two equal sides and one unequal is called isosceles and the triangle with all the unequal sides is called scalene. When mentioning this to the students, some understood that if they knew the triangles but some did not know what the sides of each of them were like.

Of course, everything can be reduced to institutionalization. Traditional teaching situations are situations of institutionalization but without the teacher dealing with the creation of meaning: it says what the child wants to know, it is explained and it is verified that he has learned it. At the beginning the researchers were a bit obsessed by the didactic situations because it was what most lacked traditional teaching Brousseau (1994) cited by Panizza (2011)

Conclusions

Based on the results of the implementation of the planned intervention activities, at the beginning of the work the idea was to look for adequate strategies to generate a significant learning and teaching of geometry. It was thought that these strategies were scarce or that there were few studies on this subject in the teaching of geometry in elementary school. And that there were few explicit ways to work this discipline.

Throughout the development of the research, the possibilities of working with geometry were expanded, with this we could make an analysis to assess, which strategies, models or theories are the most adequate to achieve the learning and teaching in the primary school of this discipline, making an intervention before the problem presented at the beginning that was supported theoretically, not only based on the empirical knowledge of the teaching practice. All of the above helped to generate a good action plan for the preparation, implementation and analysis of the results regarding the study topic.

In order to elaborate the intervention activities, the two ways of working considered the important ones in the teaching of geometry, which are the TES and the Van Hiele Model, were taken into account. When making an analysis of the results obtained from the implementation of the intervention activities. They show that in both theories it was possible to get the students to build the knowledge that was had as didactic intention as well as to develop the activities proposed by the teacher.

It should be noted that the two planning devices can be adapted for the realization of any mathematical activity and very good results can be obtained, but in comparison of the theory and the model, considering the results obtained from the analysis of said activities, concludes that there are more strengths to work geometry with the Van Hiele model. Because not only have the direct way through the phases that the student learns, but has a way of evaluation in which it is known the level of learning that the student has to start from there and thus achieve the following levels. Another very important aspect that favors learning in this model is that a subject is approached from the most general to the most specific knowledge, for example from geometry to the triangle, which greatly helps the child to better understand the content.

This does not mean that the TES does not work as a planning device for the teaching of geometric contents, but that according to the results, more strengths are denoted in the Van Hiele model. But in conclusion you can get students to build their own knowledge from the two planning devices.

Regarding the results found based on the implementation of the intervention plan, the proposed action hypothesis is confirmed: the planning devices (TES and Van Hiele) can be used to achieve the expected learning of the children. But in Van Hiele's model there will be a better understanding of the geometrical contents in the students, in addition to the fact that the proposed objectives were met.

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